## Reading Page: What is Static?

The term "static" or "static electricity" is often used when you feel a shock upon touching a metal doorknob, or when socks cling to polyester shirts in a dryer, or when your acrylic blanket creates sparks when you toss it in the dark. Static electricity was known from the times of the ancient Greeks. The word electricity comes from the Greek word elektron, which means amber. Amber is petrified tree resin, and the Greeks knew that if you rub amber with a piece of cloth, the amber attracts small pieces of leaves or dust.
"Static electricity" refers to the fact that objects acquire charge. Nature provides two kinds of charge, named by Benjamin Franklin (1706-1790) as positive and negative. All materials consist of molecules which in turn consist of atoms that are made up of positive and negative charges. The center of an atom consists of a positive, heavy nucleus with lighter negatively charged electrons surrounding it. Since atoms are neutral (under normal circumstances), they have as many positive charges as negative charges.


A "semiclassical" Bohr picture of a helium atom. The positively charged nucleus has two electrons orbiting around it

If we add electrons to an object, such as a plastic comb, it becomes negatively charged. If we remove electrons from an object, their absence makes the comb have more positive charges than electrons, so it becomes positively charged. If the object has an equal number of positive charges from the nuclei as negative charges from the electrons, it is neutral.

## Why does rubbing produce "static"?

In solid materials such as the plastic of a comb, a glass rod, or a scrap of wool, the nuclei remain more or less in fixed positions, while electrons can be removed or added easily, since the electrons are much lighter than the nucleus, and are on the outside of the atom. Some atoms (or molecules) lose electrons more easily than others do so that rubbing provides enough energy to transfer electrons from one object to another. For example, rubbing a plastic ruler with a paper towel removes electrons from the paper towel and puts them on the ruler, making the ruler negatively charged and leaving the paper towel with a deficit of electrons and thus positively charged. And since the paper towel and ruler were both neutral before they were rubbed, rubbing puts as many electrons on the ruler as it removes from the towel. Without knowing a lot more about the particular molecule that makes up an object, it is not possible to predict which object will get positively charged and which one negatively charged. Here is a table that shows how the charges separate in different materials:

| When you rub... | The charges are... |  |
| :--- | :--- | :--- |
|  | Positive |  |
| Plastic ruler with paper towel | Paper Towel | Negative |
| Rubber rod with fur | Fur | Rubber rod |
| Glass rod with silk | Glass rod | Silk |
| PVC rod with wool | PVC rod | Wool |

## How do charged objects behave?

Objects that are oppositely charged attract each other while objects that are similarly charged repel one another. If a plastic ruler is made negatively charged by rubbing it with a paper towel, and a glass rod is made positively charged by rubbing it with silk, the ruler and the rod would attract each other; however, two charged rulers would repel one another.

Separated charges frequently recombine producing a spark, for example when your charged body touches a doorknob (especially true in winter). Clouds get charged too, and layers of clouds can have different amounts of charge on them. When clouds get heavily charged, a large voltage develops between the earth and the clouds. The high voltage produced tears electrons off air molecules, providing a conducting path for the electrons. Electrons from the earth recombine with positive charges on the clouds -- causing emission of bright light, which one sees as a spark or as lightning.

You might have heard of fires in gas stations in the winter. Charge can easily build up due to rubbing clothing against the seat of a car (don't you just hate how you get zapped when you touch the handle of a car door?). If you start filling the gas tank, go back in the car, pick up some charge and come back out, the discharging spark can ignite the gasoline fumes! The same holds for filling a portable gas can inside the vehicle --it is best to put it in contact with the concrete on the ground so that any charge on the can has a chance to discharge before you start filling it.

Static electricity is not all nasty. Copying machines arrange positive charges on the surface of a nonconducting drum in the pattern of the document to be copied. Fine particles of negatively charged dry toner ink are then gently sprinkled onto the drum. The particles temporarily stick onto the drum and are later transferred onto paper and heated to "melt" them onto the paper and produce a "Xerox" copy. Electrostatic air cleaners use a similar principle to remove dust and smoke particles from the air.

## Why is there more evidence of static in the winter than in the summer?

Cold winter days are typically more dry, and you may notice that it is easier to hold charge on a rod or an electroscope on a dry day. Charged objects retain their charge for a while, after which they return to their neutral state. What, then, happened to the charge? For the most part, charge gets "removed" by water molecules in the air. Water is a polar molecule, meaning that its charge is not uniformly distributed over the molecule.

A negatively charged object attracts the positively charged end of the water molecule and gladly gives away a negative electron to the water molecule. The floating water molecule carries the electron back to a positively charged rubbing cloth, decreasing the charge on both rod and cloth. The smaller number of water molecules in the air (lower humidity) in winter helps charged objects retain their charge.


## What is "anti-static" spray and how does it work?

If one does not want a skirt or a pair of pants to get charged, one needs to conduct away the charge and take it back to the "charger," which may be a pair of socks or hose that the garment rubs against. Anti-static spray provides a thin conducting path that allows this recombination to occur. Memory chips for computers and other sensitive electronic equipment are often packed in antistatic bags too so that the fingers of a person handling it do not accidentally send a jolt of charge through the device.

## What is grounding?

"Grounding" or "connecting to earth" refers to electrically connecting an object to the earth by means of a metal wire or other conductor. Since the earth is so large and can conduct electricity, it can easily give up or accept a few electrons. The earth acts as a reservoir of charge.


When a negatively charged object is connected to earth, electrons freely flow down into the earth. Similarly, if a positively charged object is connected to the earth, there are always a few spare electrons that can flow from the earth into the object and neutralize the charge. The net effect is that an object connected to the earth becomes neutral.

Many electrical appliances are sensitive to extra charge accumulating on them. Therefore they are "earthed" or "grounded," usually through the center prong of a 3-prong electrical plug which is connected directly to earth.

Grounding is, in fact, such an efficient way of transferring charge that electricians working on a high voltage line have to be careful not to make contact with the earth by accident. If they do, the human body would provide an easy path for the charge to flow through, and the person would get electrocuted.

## Simulations:

http://phet.colorado.edu/en/simula-

tion/balloons
http://phet.colorado.edu/en/simulation/travoltage

## Reading Page: Contact Points and Light Bulbs

All electrical devices have contact points. Contact points allow electrical access into the device. They are made of metal, so electrical current has an easy path into the device. Inside the device there is a conducting path all the way - with no broken connections - so current can travel through the device.

A device must have at least two contact points. Current flows into the device through one contact point, goes through the device and out through the other contact point. Devices can have more than two contact points - examples include switches, diodes, and even bulbs. By connecting different pairs of contact points, different parts of the device are active in the circuit.

The contact points of any device must be electrically separated (or insulated) from each other. You can see a layer of black-colored epoxy that separates the two contact points in some bulbs.


## The Anatomy of Light Bulbs:

Incandescent bulb: The filament of an incandescent bulb is usually a tight coil of fine tungsten wire. When a current passes through the filament, it heats the coil and makes it white hot (about $4500^{\circ} \mathrm{F}$ or $2500^{\circ} \mathrm{C}$ ). When the filament heats up, electrical energy is converted to light and heat. Tungsten is used because it has a very high melting point. Air is removed from the bulb, or the bulb is filled with an inert gas, so that the hot tungsten does not oxidize and burn out. The two contact points of the bulb are connected to the two ends of the filament. Outside the bulb, the contact points are insulated from each other. The bulb's socket has contact points that make contact with those of the bulb. The parts of a light bulb are:
A. Glass envelope of bulb
B. Low pressure inert gas (argon, neon, nitrogen)
C. Tungsten filament
D. Contact wire 1 (between stem and left side of filament)
E. Contact wire 2 (between stem and right side of filament)
F. Support wires (which serve no electrical purpose)

G Stem (glass mount)
H. Contact wire 2 soldered onto bulb's Contact Point 2
J. Bulb Contact Point 2, sometimes called the cap or sleeve
K. Insulation
L. Bulb Contact Point 1 (rivet), soldered to contact wire 1


## Fluorescent bulb:

A fluorescent bulb consists of a glass tube filled with mercury vapor at a low pressure. Electrons boil off a hot filament and gain energy from the high voltage. These electrons and charged (ionized) gas atoms form a conducting path through the tube. The energetic electrons collide with the atoms of mercury and knock out an inner electron. An outer electron falls into the lower-energy inner level of the atom and emits ultraviolet (UV) light.

The conversion of UV to visible light occurs by a second process. UV light is absorbed by a chemical called a phosphor, which coats the inside of the tube. The UV light energizes the atoms in the phosphor and they in turn emit light of many visible colors that combine to form white light.

## Light-Emitting Diodes (or LEDs):



A diode is a device made of materials called semiconductors. Semiconductors are also used to make transistors, which are the basis of electronic devices (including cell phones and computers). Both diodes and transistors were invented in the 1950s. Silicon, germanium, and gallium arsenide are examples of semiconductors.

In order to make diodes or transistors, semiconducting crystals are grown with different types of impurities so that they have a slight excess of free electrons (n-type) or a slight deficit (p-type). Diodes are made by joining $p$-type and $n$-type semiconductor crystals to create a p -n junction diode.

While ordinary diodes or transistors are used in circuit boards, Light-emitting diodes are built to give off light of energy typical of the host semiconductor. Red, yellow, green, blue and white (by combining colors) LEDs are available. Silicon, incidentally, cannot be made into a light-emitting diode because of certain properties of its energy levels. Some common red LEDs are made using a compound called gallium aluminum arsenide.

LEDs have become cheap to make. They are commonly used as tiny indicator lights (on computer monitors, VCRs, TVs, etc). When a whole bunch of them are put together, they can be used as a light bulb. The advantage of using LEDs is that they use very little electrical energy to produce light.

## A short history of the light bulb:

- 1810, Humphrey Davy used a high powered battery to produce an electrical current between two strips of charcoal - the arc lamp.
- 1820, Warren De La Rue enclosed a platinum coil in an evacuated tube and passed a current through it - first attempt to produce an incandescent bulb.
- 1840, William Robert Grove succeeded in lighting an auditorium with incandescent lamps.
- 1841, Frederick DeMoleyns patented a bulb that used a mounted powder charcoal filament between two platinum wires in a glass bulb under vacuum.
- 1846, John Daper patented a platinum filament incandescent lamp.
- 1845, W. E. Staite, patented a second incandescent lamp.
- 1850, Edward Shepard made an incandescent light using a charcoal paper.
- 1854, Heinrich Gobel used a carbonized bamboo filament secured in a glass container for an incandescent lamp.

- 1860, John T. Way demonstrated that sending electricity through mercury vapor in a glass tube can produce light.
- 1878, Joseph Wilson Swan produced and patented a carbon filament incandescent light bulb that burned a few hours.
- 1879, Thomas Edison produced a light bulb with a filament made of carbon derived from cotton. His light bulb burned for 13.5 hours.
- 1880, Edison discovered that bamboo produced a better carbon filament, the new lamps lasted for 1200 hours.
- 1893, Heinrich Gobel received credit as the inventor of the electric incandescent lamp.
- 1906, General Electric Company patented a method for tungsten filaments for use in incandescent lamps.
- 1925, incandescent bulbs with frosted glass interiors were produced.
- 1930, photo flash light bulbs were introduced in photography.
- 1960, halogen filled incandescent lamps were introduced.
- 1991, Philips Co developed a light bulb that uses magnetic induction to excite gas to emit light (lifetime 60000 h ).
Timeline: http://invsee.asu.edu/Modules/lightbulb/history.htm


## What was Edison's role?

- He "invented" the light bulb by making vast improvements to Swan's unperfected designs.
- Edison realized that any good burner would have to have a high electrical resistance, and the need for vacuum.
- Herman Sprengel produced a device called a mercury vacuum pump, Edison bought the patent.
- While Edison held patents, invented his own light bulb, and made it into a commercially viable and working invention by extensive research and development of original ideas, he did not invent the light bulb. Instead, he bought the patents from those who did.
- U.S. Patent Office ruled, on October 8, 1883, that Edison's patents


Swan's Lamp 1879


GE's 1917 Mazda Lamp were invalid because he based them upon the earlier work of William Sawyer. To make matters worse, Swan sold his U.S. patent rights, in June 1882, to Brush Electric Company. This chain of events stripped Edison of all patent rights to the light bulb, and left him with no hope of purchasing any.

- Edison went into business setting up a direct current (DC) system of power distribution in New York City, and selling the light bulbs that used this electricity.
- He created the first practical light bulb and an electrical system to support it.



## Reading Page: Circuit Elements

## Circuit Elements:

Circuit diagrams are symbolic representations of the elements in a circuit and how they are connected. Here is a list of the circuit elements you will encounter in these activities.

| Resistor: Examples of resistors are light bulbs, ni- <br> chrome wires, pencil lead and carbon resistors. <br> The device is called a resistor. Its property is called the <br> resistance, measured in units of ohms ( $\Omega$ ). |  |
| :--- | :--- |
| Battery: A source of direct current (DC). The long stem <br> of a battery indicates the positive terminal while the <br> short stem represents the negative terminal. |  |
| Wire: A line indicates a wire that connects the ele- <br> ments of a circuit to one another. The wire is assumed <br> to have almost no resistance. |  |
| Switch: A switch is used to open or close a circuit or an <br> arm of a circuit. In the open position, a switch "breaks" <br> the circuit and does not allow current to flow in the <br> arm in which it is inserted. When closed, it completes <br> the circuit and allows current to flow. |  |
| Capacitor: A device used to store charge and then dis- <br> charge it at a later time. A camera flash stores charge <br> in a capacitor while "charging," and then quickly dis- <br> charges it through a light bulb, so you get a bright flash <br> of light. Capacitors are also used as switches in many <br> electronic circuits. |  |

For more symbols see http://www.kpsec.freeuk.com/symbol.htm

## Open and Closed circuits

A circuit is called a closed circuit (or a complete circuit) when it is connected in a loop so that battery can power the device. For example, if a bulb is connected to the battery, the bulb will light up. When one of the contacts is not electrically connected, leaving a gap in the path of the electrons, it is called an open circuit. The gap does not have to be large!


Open circuit


## Reading a Circuit Diagram:

A circuit diagram is a pictorial method of representing the elements in a circuit and showing how they are connected to each other. The easiest way to connect a circuit is to start from one point in the circuit diagram and follow a branch all the way around until you return to where you started. Then go to the next branch. Below is a detailed example:


## Reading Page: What is Charge? What is Current?

## Two Types of Charge

Charge comes in two types -- positive and negative. Positive charge resides in the nuclei of atoms. Negative charge resides in the electrons that surround the nucleus. When we remove electrons, the deficit leaves the object positively charged. When we add extra electrons, the object becomes negatively charged.

For example, when an electron is taken off a sodium atom and put on a chlorine atom, chlorine becomes negatively charged while sodium is positively charged. The attraction between the positive and negative charges binds the atoms together to make a molecule -- sodium chloride, also known as table salt.


- Charge is measured in units of coulombs (C).
- Since a single electron or proton is the smallest atomic unit of charge, all charges in nature come in multiples of this elementary charge.
- The charge of one electron is $\mathrm{e}=1.6 \times 10^{-19} \mathrm{C}$.
- A charge of -1 C contains $6.25 \times 10^{18}$ electrons.

History
In the $5^{\text {th }}$ century BCE, Greek philosophers thought all matter was composed of very small particles called atoms. "Atom" means "that which is indivisible." It was only in the early 1800s that John Dalton, an English chemist, suggested that each chemical element has a different atom.

## Components of an atom

We know now that it is possible to take atoms apart. There are three kinds of particles in an atom: the proton, which is positively charged; the neutron, which has no charge; and the electron, which is negatively charged. The protons and neutrons together form the nucleus, and occupy a small space at the center. The electrons circulate around the nucleus at a larger distance. Protons and neutrons have almost the same mass. Electrons are about 2000 times lighter. Atoms of elements are neutral: in sodium, for example, you have 11 positive charges (protons) and 11 negative charges (electrons). The number of protons in an atom determines the element and its chemical properties.

## What is current?

A current is produced when charges move. In a metal wire, the positive cores of the atoms remain fixed while the electron can move. In a metal at room temperature, the heat energy causes the electrons to move around within the metal. There is nothing to direct the motion of the electrons, so they move in random directions, bumping into anything that gets in the way - the atom cores, each other and the vibrations of the atomic nuclei - and bounce off to travel in a different direction.

To produce an electrical current, electrons must move in a specific direction. In a closed circuit the battery directs them: the negative electrons are attracted to the positive of the battery. While electrons continue to bump into stuff, the general direction of flow is toward the positive of the battery.

A large current implies that a lot of electrons flow toward the positive of the battery. Counting how many electrons flow past a given point in the wire every second would give us a measure of the electrical current. However, this number turns out to be inconveniently large. It is therefore useful
to define a more convenient unit. Since each electron has a particular amount of charge, we could measure the amount of charge that flows every second. This unit is called the ampere (A or amps).

- Current is measured in units of amperes (A or amps).
- The amount of current is given by the amount of charge that flows per sec.
- To produce a current of 1 A we need $6.25 \times 10^{18}$ electrons flowing per sec.


## Conductors and Insulators:

How easily electrons can flow in a material defines whether they are conductors or insulators. If the material has electrons that are loosely bound to the nucleus, the electrons can move easily from one atom to another - These materials are good conductors. If the electrons are tightly bound to the nucleus, and cannot move, they make good insulators.

## What is the path of current in a circuit?

## History

The convention of current flow has its origins in the history of electricity. Benjamin Franklin (1706-1790) postulated two kinds of charges, which he named positive and negative. He figured that electrical current flows from + to -, just as water flows from a high point to a low point. The carrier of current, the negatively charged electron, was only discovered in 1897 by J.J. Thompson.

In a simple one-loop circuit the electrons flow away from the negative ( - ) terminal of the battery, around the circuit, and toward the positive (+) terminal of the battery.

## Two conventions

Physicists and engineers usually consider current as flowing in the opposite direction: from the + of the battery, around the circuit to the - of the battery. Both representations are correct, so long as one sticks to one system or the other. We can distinguish them by calling them "conventional current flow" (starting from the + terminal) and "electron flow" (starting from the - terminal). This convention has its roots in Benjamin Franklin's idea of high and low potentials.


## No contradiction

Having two conventions for the flow of current may seem contradictory. However, think of this: we accept the idea of the sun rising in the east and setting in the west. But we know that the sun does not go around the earth; it is the earth that rotates in the opposite direction. Similarly, it does not matter whether we assume that conventional current flows from + to - or that electron current flows in the opposite direction. The effect is the same.

Our convention
Since students usually find it easier to imagine electrons flowing, we will use the flow of electrons to define the electron current in a circuit. The important thing is to be consistent!

## Reading Page: The Switch

All mechanical switches have:

- At least two contact points: complicated switches have more than two contact points.
- A conducting path that connects the contact points when the switch is "on." In a knife switch, a metal bar connects and disconnects the contact points to turn the switch "on" and "off."
- An insulated handle or button so the person touching the switch does not get electrocuted: lowvoltage circuits present little danger, but a 110 V household line are risky if touched.
Very fast switches, such as electronic switches used in computers, use a similar principle. Computer keyboard switches may be made with capacitors. Switches inside a computer may be made with diodes.

A Single-Pole, Single-Throw (SPST) switch, a commonly used switch, has two positions -- "on" and "off." In the "on" position it connects the two contacts with a metallic conductor and completes (closes) the circuit. In the "off" position it breaks the connection and opens the circuit. There are many different kinds of SPST switches, e.g., knife, toggle, and push-button switches.


Momentary Switches can be "normally off" or "normally on." A "normally off" switch keeps the circuit disconnected until a button is pushed (like a doorbell switch). A "normally on" switch keeps the circuit connected until the button is pushed (like a refrigerator or car-door light).

A Single-Pole DoubleThrow (SPDT) switch can be used to power two circuits in an "either circuit 1 or circuit 2" situation. It has three contact points.


A Double-Pole Single-Throw (DPST) switch has four contacts. Throwing the switch from "off" to "on" connects both the bulb and the buzzer circuits. The circuits are separate, and each has its own source of power. It is called "double pole" because the switch is like two single-pole switches side by side, but a single "throw" changes it from on to off.

## A Double-Pole Double-Throw

(DPDT) switch has six contacts. This switch is useful when one needs to reverse the direction of current in a device. For example, the circuit shown will allow a motor to spin clockwise and, at the flip of a switch, to spin counterclockwise.


## Reading Page: How Do Batteries Work?

## A battery has

- Two electrodes made of two different materials (both of which are conductors, usually metals). Zinc and carbon are frequently used as electrodes in modern dry cells.
- An electrolyte, which is a chemical such as a dilute salt solution.
- A container in which the electrolyte and electrodes undergo chemical reactions to deposit + and - charge on the electrodes.
- Two terminals, which are the parts of the electrodes that stick out of the battery. Circuits are connected to the terminals. One terminal is positive ( + ) and the other is negative ( - ).


## What happens in a battery:

- A battery separates + and - charges through chemical reactions and places them on the electrodes.
- This separation of charge produces a voltage difference between the terminals.
- The charge cannot move out of the battery until a circuit is connected between the terminals.


## Early batteries: the simple "wet" cell

Batteries work because of oxidation-reduction chemical reactions. Some substances are willing to take up extra electrons (oxidizing agents) while others are willing to give away an electron or two (reducing agents). Many good oxidizing agents are metals such as copper, gold, lead, zinc, sodium, and aluminum are good reducing agents.

A simple electric cell has copper and zinc electrodes, and a solution of a dilute salt, such as copper nitrate, as an electrolyte. Zinc dissolves as a positive ion, leaving behind two electrons on zinc electrode. The copper nitrate solution contributes positively charged copper ions (Cu++) and negatively charged nitrate ( $\mathrm{NO}_{3}-$ ) ions to the electrolyte. A porous barrier placed between the electrodes prevents copper ions from depositing on the negative zinc electrode.

When an external circuit (such as a light bulb or buzzer) is connected between the electrodes, the extra (negative) electrons leave the zinc electrode and go through the circuit to the copper


See battery animation Battery.mov electrode (since copper, the oxidizing agent, is willing to accept spare electrons). The electrons on the copper electrode attract positive copper ions to the electrode. Completing the flow of charge (inside the battery) are negative nitrate ions, which migrate through the porous barrier and neutralize positive zinc ions. This allows zinc atoms to deposit two electrons back on the electrode and become ions, making the zinc electrode negatively charged again. This cycle continues as long as the circuit is connected (until the battery dies, of course).

## The Modern Dry Cell

In a dry D or a AA cell, the electrolyte is absorbed in a powdery paste. An outer cup of zinc and an inner rod of carbon are often used as electrodes, with ammonium and zinc chloride electrolytes. Many modern batteries have manganese dioxide mixed in around the carbon rod to control the buildup of hydrogen ions that would reduce the battery voltage. Rechargeable nickel
 hydroxide-cadmium cells are used for computers and cellular phones.

## What decides the voltage of a battery?

The voltage between the terminals of a battery depends on the combination of electrodes, on their ability to dissolve or give up electrons, and on the electrolyte. The electrolyte determines the efficiency of the cell.

## Why do batteries go dead?

While the ions in the battery keep coming back to the electrode, after a while one electrode or the other dissolves and cannot maintain the chemical reaction any more. The battery goes "dead." Some batteries, such as the lead acid or nickel-cadmium battery, can be recharged by running current through them in the reverse direction. Most dry cells cannot (and should not) be recharged.

## How do batteries differ?

When we buy batteries, we usually check for voltage and size. But surely there is a difference between eight D-cells that produce 12 V and a 12 V car battery. One cannot hope to start a car with eight D-cells! The difference lies in how rapid a chemical reaction the battery's chemicals can support. Car batteries can provide large amounts of current while a little calculator battery can provide only a small current before its chemical reactions get overloaded.

While the sizes and current-producing capacities of batteries vary a lot, their voltages span a narrow range from 1.2 to 2 V . The 6 - or 9 -volt battery you can buy in the store is made of small-voltage, single-cell batteries in series (namely, connected one after another)!

## How are batteries different from a household electric outlet?

Batteries provide direct current (DC), which means that a battery's + terminal always stays + (and always stays -). An electric outlet provides alternating current (AC), which means that the terminals interchange + and - many times a second (60 times/sec in the U.S.)

A battery can provide a few volts, usually 1.5 to 9 V for most dry cells, 12 V for a car battery. An electrical outlet provides larger voltages (and therefore more energy). In the U.S. a typical outlet voltage is 110 V .220 V outlets are reserved for heavy-duty appliances such as stoves or clothes dryers. In many Asian and European countries the standard household outlet provides 220 V.

A battery produces energy via a chemical reaction. AC generators convert mechanical (e.g., hydroelectric power plants), chemical (by burning coal), or nuclear energy into electrical energy.

## Does a Battery Store a Certain Amount of Charge?

Batteries do not STORE charge or current; they GENERATE current on demand.
A battery does not allow charge to flow when not connected to a circuit. When a complete circuit is connected between the positive and negative terminals, the chemical reaction in the battery gears up and starts generating a continuous flow of charge. The rate at which the charge flows (which is the current) is dictated by how much the circuit demands, given by Ohm's law. If the demand is large, the chemical reaction proceeds faster.

There is a point beyond which the reaction cannot speed up any more. At this point the battery does not provide the full voltage since the demand has exceeded its capacity to provide charge at the needed rate.

Again, the battery is not a bin of current or energy from which appliances can eat. It generates energy on demand.

## Reference Page: Using a Multimeter

A multimeter is a device that can be used to measure several electrical characteristics (hence the name multimeter). We will use multimeters to measure resistance, current and voltage. Multimeters come in a large variety of shapes and sizes, but their characteristics are similar. Here are the important characteristics of a multimeter:

- Digital multimeters have a digital display, a knob that allows you to change between scales, and a set of terminals into which you can plug the probe wires.
- You can rotate the knob and choose between a variety of different kinds of measurements, indicated by their units: A stands for amperes, measuring current; V stands for volts, measuring voltage; $\Omega$ stands for ohms, measuring resistance.
- For all measurements, one end of the probe is plugged into the terminal marked "COM," for common. The other probe is plugged into the terminal marked with the desired unit: V or $A$ or $\Omega$. In some multimeters the same terminal may be used for more than one of these measurements.
- Within each kind of measurement, you can choose several ranges. A range, e.g., 20 m in the current range refers to $20 \mathrm{~mA}=20 \times 10^{-3}=0.02 \mathrm{~A}$. This means that you can measure between $0-20 \mathrm{~mA}$ of current in this range. A range is like a ruler - if you need to measure 40 mA , you need a longer ruler, so you will need to switch to the next higher range, namely 200 mA . Equivalences for the ranges are given below:

| Current, in amperes or A |  |  |  | Resistance in ohms or $\Omega$ |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: | :---: |
| $200 \mu$ | $200 \times 10^{-6}$ | 0.000002 | 2 k | $2 \times 10^{3}$ | 2,000 |  |  |
| 2 m | $2 \times 10^{-3}$ | 0.002 | 20 k | $20 \times 10^{3}$ | 20,000 |  |  |
| 20 m | $20 \times 10^{-3}$ | 0.02 | 200 k | $200 \times 10^{3}$ | 200,000 |  |  |
| 200 m | $200 \times 10^{-3}$ | 0.2 | 2 M | $2 \times 10^{6}$ | $2,000,000$ |  |  |
| $\mu=$ micro $=10^{-6}$ <br> $\mathrm{~m}=$ milli $=10^{-3}$ | $\mathrm{k}=$ kilo $=10^{3}$ <br> $\mathrm{M}=$ mega $=10^{6}$ |  |  |  |  |  |  |

## To measure resistance:

a) Do NOT connect the battery to the circuit when you measure resistance.Turn the selector knob on the multimeter to "resistance" or " $\Omega$."
b) Plug one lead of the multimeter into "common." Plug the other lead into " $\Omega$." Select the range of measurement (if the meter is not autoranging).
c) Check all the multimeter connections: connect the multime-
 ter leads to each other: since there is no resistance, the meter should read $0 \Omega$.
d) Connect the multimeter leads to the wires coming out of the light bulb using the alligator clip wires. Read the display.
e) If the display says 1 . or flashes, the device has a resistance larger than the range can measure. Increase the range chosen.
f) Resistance is measured in units of ohms, abbreviated " $\Omega$."

## To measure current:

Current is a measure of how many electrons flow at any point in the circuit every second. Since the number of electrons is huge (a few billion trillions/second), a different unit called amperes (or amps) is used to measure current. Here's how:
a) Connect the meter in series with the device through which you wish to measure current. Namely, connect the circuit with the multimeter in the same path as the resistor.
b) Turn the selector knob on the multimeter to "current" or "A." This makes the meter function as an ammeter.
c) Select the range of measurement using the dial or knob. If you are unsure, chose the largest range. You can always change the range during measurement.
d) Plug one of the leads coming out of the multimeter into the socket marked "common" and the other into "DC A."
e) Note: multimeters frequently have two different terminals for current. One terminal is reserved for large currents ( 10 A or 20 A ). This terminal is activated only when the dial is turned to the corresponding range ( 10 A or 20 A ). The other A terminal is used for smaller currents.


## To measure voltage:

Voltage is a measure of the change in the energy of the electron as it traverses a device. One uses the terminology "voltage across" or "voltage difference across" or "voltage dropped across" a resistor. Since we want to know the change in energy of the electron, the voltage needs to be measured between two points in a circuit, e.g., between the two ends of the device. Therefore the voltage is measured in parallel with the device.
a) Connect the circuit.
b) Turn the selector knob on the multimeter to "DC voltage" or "DC V." This makes the meter function as a voltmeter.
c) Select the range of measurement using the dial or knob. If you are unsure, chose the largest range. You can always change the range during measurement.
d) Plug one of the multimeter leads into the socket marked "common," and the other into "DC V."
e) Connect one lead of the multimeter to one contact of the device, and the other lead to the other contact.


## Common errors:

- Be sure to disconnect the battery to measure resistance. The battery should not be connected to the circuit when resistance is being measured. You run the risk of blowing the fuse of the multimeter, or your measurements will be incorrect.
- The meter should be connected in series, not in parallel, when current is being measured. If connected incorrectly, you run the risk of blowing the fuse on the meter.
- The meter should be connected in parallel, not in series, when voltage is being measured. If connected incorrectly, you run the risk of blowing the fuse on the meter.


## Reading Page: Current in Series Circuits

In a series circuit the elements (e.g., resistors) are strung one after the other. In a series circuit,

- The electric current has only a single path through the elements.
- The current passing through each of the elements in a given series string is the same.
- In the circuit shown, the current at $A$ is the same as that at B, C, D and E.


A common misconception is that the current one measures depends on where in the circuit one measures it. Students think that an ammeter will read a larger current near the battery (at A) and that the value falls off as one goes farther from the battery (say, at D). Not so! In a string of resistors in series, all resistors have the same amount of current going through them. Remember: current represents the number of electrons/second, and none of those electrons can disappear.

Drift speed: Charge travels in a wire with an average speed called the drift speed. When there is no voltage difference across a wire or resistor, electrons run around in zigzag paths, bumping into each other, atoms, and atomic vibrations. On the average they go nowhere because they pretty much run around in circles. When a voltage is applied across the resistor, the electrons still run around in zigzag paths, but they travel more toward the positive end of the resistor than toward the negative end. Thus, despite the collisions, the electrons move slowly along the resistor, gaining a small net speed toward the positive end. This net speed is called the drift speed.


The speed of electrons between collisions is very large ( $\sim 1600 \mathrm{~km} / \mathrm{sec}$ in copper). The drift speed, in contrast, is quite small, since it represents the average speed with which the electron drifts toward the positive end of the resistor. If no voltage is applied to the resistor, the drift speed is zero.

How small is the drift speed? To calculate the drift speed, one needs to know the current, the diameter of the wire, and the kind of wire being used. In the copper wire discussed in the example, the speed is about three tenths of a mm per second ( $0.3 \mathrm{~mm} / \mathrm{sec}$ ). Pretty slow! At this speed, electrons would take 56 minutes to travel 1 meter.

Why do bulbs turn on immediately? If electrons are so slow, it should take forever for a light to turn on. Not so! The electron right by the switch may take several minutes to drift over to the light bulb. However, there are electrons in the wires and the filament of the bulb, and all of them begin flowing almost immediately after the switch is turned on. There is immediate motion of electrons everywhere in the circuit and, therefore, a flow of current. The command to move travels faster than the speed at which each individual electron moves. For electrons this command travels at the speed of light ( $300,000 \mathrm{~km} / \mathrm{sec}$ ). Each individual electron, on the other hand, moves pretty slowly.

A similar phenomenon is seen in a garden hose full of water. When you open the faucet, the pressure produced makes the water squirt out of the other end. The drops of water that come out of the faucet right after you open it do not appear instantaneously at the other end of the hose. Instead, the water that is already in the hose begins moving, pushing out the water at the other end. This is also why water that comes out of a hose that has been lying on the ground on a hot summer day is hot initially, and then cools down. The "command to move" travels faster than the individual
drops of water do. For the flow of water, this pressure disturbance travels at the speed of sound. For the flow of electric current, the disturbance travels at the speed of light.

## Applications of Series Circuits:

Electrical devices are connected in series when we want the same current to flow through them. An example is a fuse or circuit breaker.

In a microwave, a fuse is placed at the point where the device plugs into the electrical outlet. All the current drawn by the various parts of the microwave has to go through the fuse. If something goes wrong and a circuit draws a lot of current, the large current "blows" the fuse, protecting the microwave from overheating.

The same idea is used in a household. A segment of the house might be all on one power line. The fuse is placed at the entry point of the outside power line. Fuses are rated for certain amounts of current, and the house is wired accordingly. If we connect several current hungry devices (such as an iron, a microwave and a curling iron) to the same circuit, perhaps drawing 15 A of current on a line with a fuse rated at 10 A , the fuse becomes hot and "blows," protecting the circuits from getting too hot, and warning the user that they need to remove a device or two from the line.

## How is a fuse constructed, and how does it "blow?"

A fuse is a thin wire (a resistor), usually enclosed in a small glass or ceramic tube. Each fuse is rated for a certain amount of current. A fuse rated at 1 A (one amp) will allow a current of less than 1 amp to flow through it. More than one amp melts the wire (the fuse "blows"). This can happen, for example, if there is a short circuit.


In most homes a circuit breaker is used in place of a fuse. Circuit breakers use bimetallic strips or magnets. A bimetallic strip is a strip made of two metals fused together. The metals are chosen to have different amounts of expansion when they heat up so the bimetallic strip bends when heated. When the current drawn exceeds the rating of the circuit breaker, it trips "off" and has to be manually reset.


# Reading Page: Resistors and Resistance 

## What is a resistor?

A light bulb is an example of a circuit element called a resistor. Other examples of resistors are the heater coils inside a toaster, a curling iron, the heating element in an electric stove, and carbon resistors that are used in electronic circuits to control the amount of current in a circuit.

A resistor resists the flow of current. A resistor is made of a conducting material that allows charge (usually electrons) to flow through it. The ease with which electrons flow is controlled by a property called resistance. The value of the resistance is a property of the resistor, and depends on its length, its diameter, and the material the resistor is made of.

| Resistance color code |  |  |  |
| :--- | :---: | :---: | :---: |
| Color | Number | Multiplier | Tolerance |
| Black | 0 | 1 |  |
| Brown | 1 | $10^{1}$ |  |
| Red | 2 | $10^{2}$ |  |
| Orange | 3 | $10^{3}$ |  |
| Yellow | 4 | $10^{4}$ |  |
| Green | 5 | $10^{5}$ |  |
| Blue | 6 | $10^{6}$ |  |
| Violet | 7 | $10^{7}$ |  |
| Gray | 8 | $10^{8}$ |  |
| White | 9 | $10^{9}$ |  |
| Gold |  | $10^{-1}$ | $5 \%$ |
| Silver |  | $10^{-2}$ | $10 \%$ |
| No color |  |  | $20 \%$ |



Symbol for resistance.
Resistance is measured in units of ohms (symbolized by the Greek letter $\Omega$, pronounced "omega").

The device is called a resistor. Its property is called resistance.


In many circuits resistors are used to control the amount of current. Printed circuit boards usually contain a large number of resistors. Resistors with a large variety of resistance values are easily available. They come in two main types: wire wound, which consist of coils of finely wound wire, and "composition," which are usually made of semiconducting carbon. A resistor is usually quite small. The value of its resistance is printed on it in a code called the resistor color code (table).

Starting from the edge of the resistor, the first two stripes represent the first two digits of the value of the resistor. The third stripe is the factor of ten by which the value should be multiplied. The fourth stripe is the accuracy or tolerance in the value.

For example, the resistor on the right has a value of 62 (blue, red) times $10^{5}$ (green) $=6,200,000 \Omega$, or $6.2 \mathrm{M} \Omega$ (megaohms) and is accurate to $10 \%$ (silver).

## What is resistance?

Electrical resistance is what it sounds like: a measure of how much a material resists the flow of current. In a metal, electrons are loosely bound to their nuclei. They cannot easily escape the chunk of metal, but within the metal they run around freely. In the process of running around, they bump into each other and into other atoms -- mainly vibrating atoms of the host material and foreign impurity atoms (e.g., oxygen in copper). The collisions deflect the electrons, making their paths zigzag around. Since there are many electrons (about ten billion trillion of them per ml), they take different paths. Although they are constantly running around they do not "get anywhere" because they go forward as much as they go backward.

When a voltage is applied to the resistor (say, by means of a battery), the positive side of the resistor attracts the electrons. They still go in zigzag paths but travel a little more toward the positive side of the resistor than in other directions. If all the electrons swooped toward the positive side of the resistor without anything to bump into, the flow of electrons would be very large: it would seem as if nothing resisted the flow of the electrons. However, since there are things to bump into, the electrons still take a zigzag path. This bumping of electrons with vibrating atoms, defects and impurities in the metal is what causes resistance. The collisions within the wire are similar to an internal friction or drag.

As electrons bump into other atoms they lose energy. The energy lost by the electrons heats up the resistor. If a resistor gets very hot, it glows, producing light. In a light bulb the resistor is usually made of tungsten wire, which can get quite hot. To keep this hot wire from oxidizing, the air inside of the light bulb is removed and replaced with argon gas.

Here is an analogy of resistance: imagine a long corridor with many people standing in an wellordered grid. A child tries to run through the corridor. She winds up bumping into people, and the more people she bumps into, the harder it seems to negotiate the distance. An electron behaves similarly as it tries to negotiate a crystal of atoms.

## Does resistance change with temperature?

Yes, it does. When a resistor gets hot, its resistance increases. The resistance of a hot light bulb can be nearly ten times that of a cold one. Thus, if you measure the resistance of a light bulb when it is cold, then light it up by connecting it to a battery and evaluate its resistance when it is hot, the two values of resistance will be different because the resistance has increased with temperature.

## Reading Page: The Resistance of a Wire

## Determining resistance

The resistance of a wire depends on three parameters:

- Resistivity of the material
- Length of the wire
- Cross-sectional area of the wire

Several atomic parameters are lumped into the property called resistivity. These include the time an electron can travel without bumping into something and the number of
 electrons in the material per unit volume. Resistivity also depends on the temperature and the material's purity.

The dimensions of a wire also determine its resistance: a skinny wire constricts the flow of electrons just as a skinny tube makes water flow slowly. Furthermore, the longer the wire, the greater its resistance.

## Implications

- A good electrical conductor has a low resistivity.
- All good conductors are not equal: silver is the best, but copper is close, and aluminum is only twice as resistive. Copper and aluminum wire are usually used for wiring homes.
- The resistance of a wire is not determined by its

The resistance of a wire is given by:
Resistance $=\frac{\text { resistivity } x \text { length of wire }}{\text { cross sectional area }}$ or

$$
R=\rho \frac{L}{A}
$$

where
$\mathrm{R}=$ resistance in units of $\Omega$
$\rho=$ resistivity in $\Omega \mathrm{cm}$
$\mathrm{L}=$ length of resistor in cm
$A=$ cross sectional area of resistor in $\mathrm{cm}^{2}$ ( $\rho$ is a Greek letter pronounced 'rho') resistivity alone. A wire made of a good conductor like copper can have a high resistance if it is long and skinny.

- The resistance of a wire determines how much current it can carry. If the resistance is large, it will heat up and cause a fire hazard. The National Electrical code defines recommendations for

| American Wire Gauge (AWG) Table |  |  |
| :---: | :---: | :---: |
| Gauge <br> number | Diameter at <br> $20^{\circ} \mathrm{C}(\mathrm{mm})$ | Ohms per 1000 m <br> at $20^{\circ} \mathrm{C}$ (copper) |
| 10 | 2.58 | 1.1 |
| 12 | 2.05 | 1.6 |
| 14 | 1.63 | 2.5 |
| 16 | 1.29 | 4 |
| 18 | 1.02 | 6.4 |
| 20 | 0.81 | 10.2 |
| 22 | 0.64 | 16.1 |
| 24 | 0.51 | 25.7 |
| 26 | 0.40 | 40.8 |
| 28 | 0.32 | 64.9 |
| 30 | 0.25 | 103.2 |
| 32 | 0.20 | 164 |
| 34 | 0.16 | 261 |
| 36 | 0.13 | 415 | the sizes of wires and the current they can carry.

## Sizes of Wires: Selection from the American Wire

 Gauge TableWire diameters are defined by several different kinds of gauges. The American Wire Gauge (AWG) is the commonly used standard in the US.

Thicker wires have smaller gauge numbers. Wire gauges of 20 to 24 are convenient for the circuits in this unit. Wires of smaller gauge numbers are thick and inconvenient for this unit. Gauges 10-14 are commonly used in household wiring since they can carry a larger current.

Wires are available as single strand (solid) or multistrand. The gauge number relates to the total diameter of the conductor (single or multi-strand, without insulation). In a multi-strand wire it does
not matter if the individual wires are skinnier. Multi-strand wires are more flexible and easier to solder..

Wires used to hook up a circuit are assumed to have a very low resistance. However, this depends on the kind of metal from which they are made (which defines your resistivity), as well as on the diameter and length.

In the first example below we calculate the resistance of a short piece of hookup wire. In the second example we compare the resistance of thick and thin wires.

Example 1: Calculate the resistance of a piece of 22-gauge copper hookup wire that is $15 \mathrm{~cm}\left(6^{\prime \prime}\right)$ in length.

Solution:

Resistance $\quad R=\rho \frac{L}{A}$
where

| Resistivity of various materials at $\mathbf{2 0}^{\circ} \mathbf{C}$ in <br> units of $\Omega \mathbf{~ c m}$. <br> Conductors: |  |
| :--- | :--- |
| Silver | $1.59 \times 10^{-6}$ |
| Copper | $1.68 \times 10^{-6}$ |
| Gold | $2.44 \times 10^{-6}$ |
| Aluminum | $2.65 \times 10^{-6}$ |
| Tungsten | $5.6 \times 10^{-6}$ |
| Iron | $9.71 \times 10^{-6}$ |
| Platinum | $10.6 \times 10^{-6}$ |
| Mercury | $98 \times 10^{-6}$ |
| Nichrome* | $100 \times 10^{-6}$ |
| Semiconductors: |  |
| Graphite (carbon) | 3 to $60 \times 10^{-7}$ |
| Germanium | 1 to $500 \times 10^{-5}$ |
| Silicon | 0.1 to 60 |
| Insulators: |  |
| Glass | $10^{11}$ to $10^{13}$ |
| Hard rubber | $10^{15}$ to $10^{17}$ |
| Quartz | $75 \times 10^{18}$ |
| *an alloy of nickel, iron and chromium |  |

$\rho=$ resistivity of copper, $1.68 \times 10^{-6} \Omega \mathrm{~cm}$
$\mathrm{L}=$ length of resistor, 15 cm
A = cross-sectional area of resistor in $\mathrm{cm}^{2}$
This gauge of wire has a diameter 0.0645 cm (see table). Therefore its radius $r$ is half as much, $r=$ 0.0323 cm . The cross-sectional area is, therefore,

$$
\begin{aligned}
A & =\pi r^{2} \\
& =\pi(0.323 \mathrm{~cm})^{2}=0.0033 \mathrm{~cm}^{2}
\end{aligned}
$$

Substituting,

$$
\begin{aligned}
R & =\frac{\left(1.68 \times 10^{-6} \Omega \mathrm{~cm}\right)(15 \mathrm{~cm})}{0.0033 \mathrm{~cm}^{2}} \\
& =0.008 \Omega
\end{aligned}
$$

Unit Check:

$$
\begin{aligned}
& R=\rho \frac{L}{A} \\
& {[\Omega]=[\Omega \mathrm{cm}] \frac{[\mathrm{cm}]}{\left[\mathrm{cm}^{2}\right]}} \\
& {[\Omega]=[\Omega \mathrm{cmz}] \frac{[\mathrm{cm}]}{\left[\mathrm{cmm}^{2}\right]}} \\
& {[\Omega]=[\Omega]}
\end{aligned}
$$

The resistance offered by a small length of copper wire used to hook up bulbs in a circuit is close to zero as compared with the resistance of a bulb which is typically a few ohms.

Example 2: Calculate and compare the resistances of two 120 meter lengths of copper wire to be used in wiring a home: one is a 22 -gauge wire, and the other a 14 -gauge wire. See table for diameters.

Solution:
Resistance $=$ resistivity x length $/$ cross sectional area, or

$$
R=\rho \frac{L}{A}
$$

where
$\rho=$ resistivity of copper, $1.68 \times 10^{-6} \Omega \mathrm{~cm}$
$\mathrm{L}=$ length of resistor, $120 \mathrm{~m}=12000 \mathrm{~cm}$
A = cross-sectional area of wire 3 in $\mathrm{cm}^{2}$
The diameter of the 22 gauge wire is 0.0645 cm ; radius $r=0.0323 \mathrm{~cm}$. The cross sectional area is

$$
A=\pi r^{2}=\pi(0.0323)^{2}=0.0033 \mathrm{~cm}^{2}
$$

Substituting,

$$
R_{22 \text { gauge }}=\frac{\left(1.68 \times 10^{-6} \Omega \mathrm{~cm}\right)(12000 \mathrm{~cm})}{0.0033 \mathrm{~cm}^{2}}=6.2 \Omega
$$

The diameter of the 14 -gauge wire $=0.163 \mathrm{~cm}$; radius $r=0.0815 \mathrm{~cm}$. The cross-sectional area is

$$
A=\pi r^{2}=\pi(0.08)^{2}=0.02 \mathrm{~cm}^{2}
$$

Substituting,

$$
R_{14 \text { gause }}=\frac{\left(1.68 \times 10^{-6} \Omega \mathrm{~cm}\right)(12000 \mathrm{~cm})}{0.02 \mathrm{~cm}^{2}}=1 \Omega
$$

From this calculation we see that a 120 m length of 22-gauge wire has a resistance comparable to a small Christmas light bulb and would heat up if it carried much current. In contrast, the 14-gauge wire has a resistance that is about six times smaller, and would not get as hot (the heating is proportional to the power used and, for the same current going through the wire, increases as the resistance increases).

## Reading Page: What is Voltage?

In lay terms, voltage provides the "push" that makes electrons flow. This "push" is provided to the electrons by the battery.

Voltage is a measure of electrical energy (also called electrical potential energy). It is tricky to measure the energy of an electron in absolute terms, but it is simpler to measure the energy difference between two points. That's what a voltmeter does - it compares the energy between two points on the circuit, and shows us how much energy was "lost" by an electron as it flowed from point $X$ to point $Y$. The energy was not really lost - the electrical energy was just converted to heat and light, but from the point of view of the electron, it now has less energy!

The electrical energy lost by the electron can be shown graphically. The circuit on the right shows two resistors of the same resistance in a series circuit.


Location in circuit

It is important to realize that an electron in this circuit loses energy only when it has to traverse an electrical device that has resistance. Therefore, it does not lose energy as it travels along a wire, which has almost zero resistance. It only loses energy through resistors, which have significant resistance. Let's trace the electron's path and its energy loss:

- As the electron goes through the battery from + to - it gains energy equivalent to the voltage of the battery ( 6 V for the circuit shown).
- At the negative (-) of the battery it has an energy of 6 V . These 6 V worth of energy is all that the electron can lose in the circuit, and it distributes its energy loss over the resistors.
- Since all we have is wire from the $(-)$ of the battery to $B$, the electron's


Voltage difference between various points in the circuit
 energy stays 6 V from ( - ) through A to B.

- While traversing the first bulb, which has a resistance, the electron loses energy between B and C. The electrical energy lost by the electron is converted to light and heat. Since both bulbs have the same resistance, half of the energy lost in the circuit, namely 3 V , is lost at the first bulb.
- As the electron travels through the wire from $C$ to $D$, the electron does not lose any more energy, and its energy stays at 3 V .
- At the second bulb, the electron loses energy between D and $\mathrm{E}, 3 \mathrm{~V}$ again.
- No energy is lost during the electron's travel from E through F.
- The electron arrives at the (+) of the battery with no electrical potential energy. The battery acts like a pump and gives it 6 V of energy. Emerging from the $(-)$ of the battery, it is ready to go around the circuit again.
- When you measure the voltage difference across the first bulb (between $B$ and $C$ ), you measure the energy difference: $6 \mathrm{~V}-3 \mathrm{~V}=3 \mathrm{~V}$; across the second resistor ( D to E ), the meter reads $3-0=3 \mathrm{~V}$. Hence the phrase "voltage drop across a resistor."
- When you measure the voltage difference across both bulbs together ( $B$ to $E$ ), you get $6-0=6 \mathrm{~V}$. This makes sense, since $B$ and $E$ are directly connected to the battery!
- Each electron that goes through the circuit loses electrical potential energy as it travels through a resistor.
- The amount of potential energy lost = (the voltage drop in volts $x$ amount of charge in coulombs).
- In order to make charges flow continuously there has to be a sustained difference in voltage difference across the resistor. This is provided by a battery or a generator.


## Measuring Voltage <br> Using a Multimeter as a Voltmeter

Problem: A $5 \Omega$ resistor is connected to a 7 V battery. Draw the circuit and use the picture of a blank multimeter to show how you measure the voltage across the resistor.

Calculation: The circuit is drawn below. Since the voltage of the battery is dropped across just the one resistor, the meter should measure 7 V .
4. The meter then
reads $+7 V$.
3. Set the dial of the meter to the $D C$ volts side. Choose the appropriate range (for this problem, $0-20 \mathrm{~V}$, since we need to measure 7 V).
2. Connect the contact of the resistor closer to the positive of
 battery to the "Volts" or "V" terminal of meter.


Ohm's law is named after Georg Simon Ohm (17871854), a high school teacher in Köln and later a professor in Munich, Germany. He formulated the concept of resistance and discovered the proportionality that goes by his name. Ohm's law relates the voltage, current, and resistance in a circuit. Ohm's law states that the voltage difference across a resistor equals the current flowing through it times the resistance:

Voltage $=($ Current $) \times($ Resistance $)$
or symbolically as
$V=I R$
where
$\mathrm{V}=$ voltage in volts ( V )
$I=$ current in amperes (A)
$\mathrm{R}=$ resistance in ohms ( $\Omega$ )
In units Ohm's law can be written as
volts $=$ amps $\times$ ohms, or $[\mathrm{V}]=[\mathrm{A}] .[\Omega]$
Simulations: http://phet.colorado.edu/simulations/ sims.php?sim=Circuit_Construction_Kit_DC_Only

## Implications:

- Ohm's law tells us that for a given resistor, the greater the voltage, the greater the current.
- The resistor dictates the amount of current that flows. For the same battery voltage, a large resistance draws a small current, while a small resistance draws a large current.
- Ohm's law applies to resistors. Devices for which the applied voltage and current are in directproportion to one another are called "ohmic" or "linear devices."
- The current-voltage graph for an ohmic device is a straight line, as shown in the top graph.
- Many devices, such as capacitors, diodes, and solenoids do not behave in an ohmic manner. The current-voltage graph for a non-ohmic device, a light-emitting diode, is shown in the bottom graph. Notice that its current-voltage graph is not a single straight line over the entire range of measurement.
- Ohm's law is useful in figuring out any one of the three quantities, $\mathrm{V}, \mathrm{I}$, or R , if you know the other two. It can be applied to individual resistors, to strings of resistors, or to an entire circuit.


Current-Voltage curve for an ohmic material. The slope gives the $1 /$ resistance of the resistor. (Note that current is plotted on the vertical axis in this graph)


Current-Voltage curve for a non-ohmic device such as a light-emitting diode. Note that the curve is not a straight line all through, but only above a certain voltage. The region above 1.8 V is called the ohmic region.

- Are light bulbs ohmic devices? Light bulbs are not the best examples of resistors because your resistance changes with temperature. However, at any particular temperature they do obey Ohm's Law.
- The slope of the I-V graph (rise/run) gives us a value that has units of $[\mathrm{A}] /[\mathrm{V}]$. From Ohm's Law, we see that these units translate to $1 /$ resistance, in units of $1 / \Omega$.
- Note that this data plots current on the vertical and voltage on the horizontal - different from the axis used in the Figuring out a law lab. Current- voltage graphs are frequently drawn in this manner, and one has to remember that the slope gives $1 / R$ rather than $R$.

Example 1: In a single-bulb circuit, the voltage of the battery is 6 V , and the resistance of the light bulb is $4 \Omega$. Calculate the current.

## Solution:

$\mathrm{V}=6 \mathrm{~V} ; \mathrm{R}=4 \Omega ; \mathrm{I}=$ ?
Ohms' Law gives us

$$
\begin{aligned}
& V=I R \\
& (6 \mathrm{~V})=I(4 \Omega) \\
& I=\frac{6 \mathrm{~V}}{4 \Omega}=1.5 \mathrm{~A}
\end{aligned}
$$

The current through the resistor is 1.5 A .
Example 2: In a single-bulb circuit, the battery has a voltage of 24 V , and the current passing through the bulb is 0.02 A . Calculate the resistance of the bulb.

## Solution:

$\mathrm{V}=24 \mathrm{~V} ; \mathrm{I}=0.02 \mathrm{~A} ; \mathrm{R}=$ ?
Ohms' Law gives us

$$
\begin{aligned}
& V=I R \\
& (24 V)=(0.02 A) R \\
& R=\frac{24 V}{0.02 A}=1200 \Omega
\end{aligned}
$$

The resistance of the resistor is $1200 \Omega$.
Summary

|  | You measure | Symbol | Units |
| :--- | :--- | :--- | :--- |
| Electrical Current | Intensity of elec- <br> tron flow | I | A (amp) |
| Resistor | Resistance | R | $\Omega$ (ohm) |
| Battery | Voltage | V | V (volt) |

## Reading Page: Ohm's Law and Series Circuits

Sample Problems using Ohm's Law in Series Circuits.
Example 1: Three resistors of resistances of $4 \Omega$ each are connected in series. A 6 V battery is connected across them.
a) Calculate the total resistance of the circuit.
b) Calculate the current through the resistors.
c) Is the current through each resistor different? Or the same? Explain your reasoning.
d) Calculate the voltage across each resistor.
e) Draw a graph of the voltage at various points in the circuit.

## Solution:

Battery voltage $\mathrm{V}=6 \mathrm{~V}$;
Resistance $R_{1}=R_{2}=R_{3}=4 \Omega$
(a) Total resistance:
$R_{s}=4+4+4=12 \Omega$.
(b) Current $\mathrm{I}=\mathrm{V} / \mathrm{R}_{\mathrm{s}}=6 / 12=0.5 \mathrm{~A}$.

(c) Since the resistors are in series, all resistors have the same current of 0.5 A going through them.
(d) Using Ohm's Law, the voltage difference across the $R_{1}$ is $\mathrm{V}_{1}=I \mathrm{R}_{1}=0.5 \times 4=2 \mathrm{~V}$.
$\mathrm{V}_{2}=\mathrm{IR}_{2}=0.5 \times 4=2 \mathrm{~V}$.

$V_{3}=I R_{3}=0.5 \times 4=2 \mathrm{~V}$.
(Seem like a long way to do this problem? The technique is handy when the resistors are different.)

Example 2: Two resistors with resistances $R_{1}=10 \Omega$ and $R_{2}=5 \Omega$ are connected in series across a 6 V battery.
a) Calculate the total resistance of the circuit
b) Calculate the current in the circuit.
c) Calculate the voltage across each resistor.
d) Draw a graphical representation of the voltage at different points in the circuit.

## Solution:

Battery voltage $\mathrm{V}=6 \mathrm{~V}$
(a) Total resistance $\mathrm{R}_{\mathrm{s}}=10+5=15 \Omega$
(b) Current I $=\mathrm{V} / \mathrm{R}_{\mathrm{s}}=6 / 15=0.4 \mathrm{~A}$
(c) Each resistor has a current of 0.4 A going through it. Using Ohm's Law, the voltage difference across the $R_{1}$ is
$\mathrm{V}_{1}=\mathrm{IR}_{1}=0.4 \times 10=4 \mathrm{~V}$.
And the voltage difference across $R_{2}$ is
$V_{2}=I R_{2}=0.4 \times 5=2 \mathrm{~V}$.

The $10 \Omega$ resistor had twice as much voltage across it as the $5 \Omega$ resistor.


Example 3: Four bulbs of resistance 4, 8, 6 and $5 \Omega$
The total voltage across both resistors $=4+2=6$ V. (when hot) are connected in series. When the bulbs are connected to a battery, a current of 1.5 A flows through the circuit..
a) Would you expect the current through each of the bulbs to be different? Explain your reasoning.
b) Calculate the total (or equivalent) resistance of the circuit.
c) Calculate the voltage of the battery.
d) Would you expect the voltage across each of the bulbs to be different? Or not? Explain your reasoning.
e) Calculate the voltage across each of the bulbs using Ohm's law.
f) Use the picture of the blank multimeter and connect it appropriately to measure the voltage cross the $6 \Omega$ resistor.

$\mathrm{R}_{1}=4 \Omega ; \mathrm{R}_{2}=8 \Omega ; \mathrm{R}_{3}=6 \Omega ; \mathrm{R}_{4}=5 \Omega ; \mathrm{I}=1.5 \mathrm{~A}$;
(b) $R_{s}=4+8+6+5=23 \Omega$
(c) The battery's voltage is dropped across all the resistors together, of total resistance $R_{s}=23 \Omega$.

Therefore,

$$
V=I R_{s}=1.5 \times 23=34.5 \mathrm{~V} .
$$

(d) The voltage difference across each bulb will be different. They all have the same current flowing through them (since they are in series), the larger resistor will need to have more voltage difference across it than a smaller resistor does.
(e) $\mathrm{V}_{1}=I \times R_{1}=1.5 \times 4=6 \mathrm{~V}$
$\mathrm{V}_{2}=1 \times \mathrm{R}_{2}=1.5 \times 8=12 \mathrm{~V}$
$V_{3}=I \times R_{3}=1.5 \times 6=9 \mathrm{~V}$
$V_{3}=I \times R_{4}=1.5 \times 5=7.5 \mathrm{~V}$
(f) See picture.

Example 4: Six resistors, each of resistance $4 \Omega$ are connected in series to a battery. A current of 1.4 A flows through the first bulb.
a) How much current flows through the other bulbs? Explain you reasoning.
b) Calculate the voltage of the battery.

## Solution:

$R_{1}=R_{2}=R_{3}=R_{4}=R_{5}=R_{6}=4 \Omega ; I=1.4 \mathrm{~A}$.
(a) Because all bulbs are in series, the same amount of current, 1.4A , flows through all the other bulbs as well.
(b) To calculate the voltage of the battery: the entire battery voltage is distributed across all the resistors, whose total resistance is
$R_{s}=R_{1}+R_{2}+R_{3}+R_{4}+R_{5}+R_{6}=24 \Omega$.
The voltage of the battery is therefore $\mathrm{V}=1 \mathrm{R}_{\mathrm{s}}=1.4 \mathrm{X} 24=33.6 \mathrm{~V}$.

Example 5: A circuit with four bulbs of equal resistance is connected in series. When a 9 V battery is connected across them, a current of 0.6 A flows through the circuit.

Calculate the resistance of each bulb.

## Solution:

Method 1: Applying Ohm's Law to the whole string
Because we have a series circuit, all bulbs have the same current flowing through them, $\mathrm{I}=0.6 \mathrm{~A}$. The total resistance of the four bulbs determines the amount of current. If each of the bulbs has a resistance of $r$, the combined resistance of the four bulbs is $4 r$. The battery voltage is 9 V across the four-bulb string.


Thus, $R=4 r, V=9$ volts, and $I=0.6$ amps.
Substituting into $V=I R$ we get
$9=(0.6) 4 r$

Rearranging this equation for the resistance of each bulb:
$r=\frac{9}{(0.6) 4}=3.75 \Omega$

Method 2: Applying Ohm's Law to each bulb in the string
An alternate way to do this problem is to find the voltage difference across each bulb in the circuit and then calculate the resistance of a single light bulb using Ohm's Law:

Because all the bulbs have the same resistance ( $r$ ), the voltage difference across each bulb must be the same. Since there are four bulbs and the battery voltage is 9 V , each bulb has a voltage of $9 / 4=2.25 \mathrm{~V}$ across it.

Thus, each bulb of resistance $y$ has voltage of $\mathrm{V}=2.25 \mathrm{~V}$ across it, and current of $\mathrm{I}=0.6 \mathrm{~A}$ flowing through it. Ohm's Law for one bulb is $(2.25)=(0.6) I$ which can be rearranged to calculate $r$ :
$r=\frac{2.25}{0.6}=3.75 \Omega$
This method gives the same answer, as expected. Both methods are valid (and equivalent).


Example 6: In a three-bulb series circuit, one bulb has a certain resistance $r$, the second bulb has twice that resistance, and the third has three times the resistance of the first. They are connected in series to a 12 V battery, and a current of 0.4 A flows through the circuit. Calculate the resistance of each bulbs.

Method 1: Applying Ohm's Law to the whole string
Resistance of bulb $1=r$
Resistance of bulb $2=2 r$
Resistance of bulb $3=3 r$
Because this is a series circuit, the total resistance of the three-bulb string is equal to the sum of the individual resistances:
$R_{s}=r+2 r+3 r=6 r$
The total battery voltage $\mathrm{V}=12 \mathrm{~V}$ is across the three-bulb string. The current in the circuit is $\mathrm{I}=0.4 \mathrm{~A}$

Applying Ohm's Law,
$V=I R_{s}$
Bulbs 1, 2, and 3 have resistances of $5 \Omega, 10 \Omega$ and $15 \Omega$, respectively.

## Method 2: Applying Ohm's Law to one bulb

Applying this method is a little trickier than when all bulbs had the same resistance. Since the resistances are multiples of $r$, we can figure out the voltage difference across each bulb in the following manner.

Because all bulbs are in a series string, they will have the same current flowing through them, $\mathrm{I}=$ 0.4 A . The larger resistance will have a larger voltage difference across it (Ohm's Law requires that V be proportional to $R$ if the current is the same). Thus the voltage difference across each bulb will be

Voltage $V_{1}=v$ across bulb 1 of resistance $r$
Voltage $V_{2}=2 \mathrm{v}$ across bulb 2 of resistance $2 r$
Voltage $V_{3}=3 \mathrm{v}$ across bulb 3 of resistance 3 r
The total voltage difference across the three resistors is: $v+2 v+3 v=6 v$ which should be equal to the battery voltage.

Thus $6 \mathrm{v}=12$ volts so that $\mathrm{v}=2$ volts.
Thus bulb 1 of resistance $r$ has 2 volts across it and a current of 0.4 A flowing through it.
Applying Ohm's Law,
$\mathrm{V}_{1}=\mathrm{Ir}$ gives
$2=(0.4) r$
$r=\frac{2}{0.4}=5 \Omega$
The resistance of bulb 1 is $5 \Omega$. Therefore bulbs 2 and 3 have resistances of $10 \Omega$ and $15 \Omega$, respectively.

## Reading Page: Parallel Circuits

Devices are connected in parallel when the left contacts of the devices are connected together, and the right contacts are connected together. Or, you could say that the devices are connected between the same two points in a circuit. Of the devices we have seen, resistors can be connected in parallel; batteries can also be connected in parallel.

## Comparing currents in one-bulb and two-bulb parallel circuits:

In a parallel circuit the voltage difference across all the resistors that are connected in parallel is the same. If these resistors are connected directly to the battery, as in the figures on the right, the voltage across each of the resistors is equal to the the voltage of the battery. The battery has to provide each resistor its rightful share of current in accordance with Ohm's law.

- If there is only one resistor $\mathrm{R}_{1}$ in the circuit, a certain current $I_{1}$ flows in the circuit. The voltage of the battery and the resistance of the resistor determine this current (Ohm's law): $\mathrm{I}_{1}=\mathrm{V} / \mathrm{R}_{1}$.
- If we add a second resistor $\mathrm{R}_{2}$ in parallel, the same voltage V is seen by the second resistor. A current $\mathrm{I}_{2}$ flows in the second branch, where $\mathrm{I}_{2}=\mathrm{V} / \mathrm{R}_{2}$. The current $I_{2}$ is independent of $I_{1}$ in the first branch.
- The battery has to supply a current $\mathrm{I}=\mathrm{I}_{1}+\mathrm{I}_{2}$.
- If a third resistor $R_{3}$ is added in parallel, the battery has to provide even more current, $I=I_{1}+$ $I_{2}+I_{3}$.
- As we add more resistors in parallel, the battery has to provide more and more current.
- An analogous situation occurs when a river splits around an island. The flow rate of water before the island is $I$, and $I$ splits into $I_{1}$ and $I_{2}$ as it flows
 around the island, where $I=I_{1}+I_{2}$. The two upper and lower branches join back together on the other side of the island, producing the original flow rate of I again.


## Representing the current in parallel circuits graphically:

- In a circuit where all the resistors are in parallel, the total current provided by the battery is equal to the sum of the currents in the individual branches.
- The current splits into two parallel branches, flows through the resistors, recombines on the other side, and flows back to the battery.
- The current and voltage by location in the circuit is shown in the figure.
- The amount of current is still decided by Ohm's Law - the battery voltage and the resistance decides the current through each branch.
- In the top figure, the path of the current is shown. A current, I, flows from the negative of the battery, (-), through A and up to B. Since the circuit now has two paths, some of the current takes the top path (B-C-D-G), and the rest the bottom path (B-E-F-G). If the two resistors are identical, equal amounts flow in the top and the bottom branches.
At G , the flow from the two paths join again, and the original current I flows from G to H to (+).
- The graph of the current at different locations in the circuit is shown in the middle diagram. Current I flows from - to A to B, then splits into two paths, the top path along C-D, the bottom along E-F. The currents join again at G to form the original current I, which goes on to flow from G to H to $(+)$.


Current vs. location in the circuit
 affect it. Therefore, the full battery voltage appears across the bottom resistor between E and F.

## Batteries in series and parallel:

## Batteries in series:

When batteries are hooked up so that the positive of the first is connected to the negative of the second and so on, they are in series.

When batteries are in series, the voltages of the individual batteries add up. If the individual batteries had voltages of $\mathrm{V}_{1}, \mathrm{~V}_{2}, \mathrm{~V}_{3} \ldots$ the total voltage of the group of batteries would be

$$
V=V_{1}+V_{2}+V_{3}+\ldots \ldots
$$

For example, each battery produces 4 V . Three batteries in series produce $3 \times 4=12 \mathrm{~V}$.
This arrangement is used to get a large voltage using several batteries of small voltage. Devices such as flashlights, cameras, radios, and calculators use this arrangement.

## Batteries in parallel:

When batteries are hooked up with the positive terminals connected to each other and the negative terminals connected to each other, and with the circuit connected between the positive and the negative, the batteries are in parallel.

When batteries are in parallel, the voltage supplied is that of one battery. The total current drawn by the circuit is split among the batteries, so each battery needs to provide only a part of the current.

Example: Three batteries, each of which can produce 8 V are connected in parallel. The circuit to which they are connected, which sees a voltage of 8 V , demands a current of 6 A - thus, each battery only needs to supply a current of 2 A .

This arrangement is used when the voltage of the battery is adequate but a large current is required, larger than what one battery alone could provide.

- If the circuit needs a lot of voltage but does not demand much current -- stack batteries in series.
- If the circuit does not need much voltage but demands a lot of current -- connect batteries in parallel.
- If the circuit needs a large voltage and demands a large current -- make several stacks of the batteries in series and then connect them in parallel.


## Reading Page: The Resistance of a Parallel Circuit

## Why is the resistance of a parallel circuit less than that of each individual resistor?

When we connect two resistors in parallel, we provide two paths for the current. The electrons now encounter less resistance - similar to providing a thicker wire, which decreases the resistance.

An alternative way is to look at the current provided by the battery.

- The battery does not know if there are two resistors or twenty. All it knows is that the total resistance demands a current I.
- Since the current demanded by the parallel circuit is more than that for one bulb, the battery thinks the circuit resistance is less.
- If one were to replace the resistance of the two-bulb tree with just one resistor that pulls the same current $I$, that resistor should have a resistance less than either $R_{1}$ or $R_{2}$.
- As we add resistors in parallel, the battery has to provide more and
 more current -for the battery, the circuit's resistance has decreased.

Figuring the resistance of a parallel circuit:

- Resistors that are connected in parallel have a total (or equivalent resistance) given by a formula that is a bit more complicated than for the series circuit. This equivalent resistance of a parallel circuit, $\mathrm{R}_{\mathrm{p}}$ is given by:
$\frac{1}{R_{P}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}+\frac{1}{R_{3}}+\ldots .$.
- Pictorially, the equivalent resistance is the value of a single resistor that will demand the same current when the same voltage is applied to the circuit.


Example 1: Three resistors of resistances $R_{1}=3 \Omega, R_{2}=4 \Omega$, and $R_{3}=6 \Omega$ are connected in parallel. What is the total resistance of the circuit?

$$
\begin{array}{ll}
\frac{1}{R_{P}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}+\frac{1}{R_{3}} & \frac{1}{R_{P}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}+\frac{1}{R_{3}} \\
\frac{1}{R_{P}}=\frac{1}{3}+\frac{1}{4}+\frac{1}{6} & \frac{1}{R_{P}}=\frac{1}{3}+\frac{1}{4}+\frac{1}{6} \\
\frac{1}{R_{P}}=0.33+0.25+0.17 & \frac{1}{R_{P}}=\frac{4+3+2}{12} \\
\frac{1}{R_{P}}=0.75 & \frac{1}{R_{P}}=\frac{9}{12} \\
R_{P}=\frac{1}{0.75}=1.33 \Omega & \frac{1}{R_{P}}=\frac{3}{4} \\
R_{P} & =\frac{4}{3}=1.33 \Omega
\end{array}
$$

Solution: two methods of doing the calculation, both of which give an equivalent resistance of $1.33 \Omega$.

Caution: Remember that the equation gives the value of $\frac{1}{R_{P}}$ and not $R_{p}$ ! Remember to invert to get
$\mathrm{R}_{\mathrm{p},}$ as in the last step!! $R_{p}$, as in the last step!!

Curious fact: Notice that the resistance of the entire circuit is less than that of the individual resistors. This might seem strange, given the fact that you are adding resistors to the tree! This phenomenon occurs because each resistor in a parallel circuit forms its own individual closed circuit with the battery and makes its own demand for current. As one adds resistors in parallel, more current is demanded of the battery. From the point of view of the battery, it provides more current at the same voltage, which means (to it) less resistance! It all goes back to Ohm's Law.

Example 1: An electric dryer on a 220 V power line has a resistance of $18 \Omega$. If an electric stove of resistance $15 \Omega$ were placed on the same line, how much total current would both of them together draw?

## Solution:

Appliances are always connected in parallel to the power source (as also bulbs, electric outlets, etc.). See diagram.

Dryer: $R_{1}=18 \Omega$; Stove: $R_{2}=15 \Omega$.
Voltage of power source, $\mathrm{V}=220 \mathrm{~V}$ across each appliance
The current drawn by the dryer, $\mathrm{I}_{1}=\mathrm{V} / \mathrm{R}_{1}=220 / 18=12.2 \mathrm{~A}$
The current drawn by the stove, $\mathrm{I}_{1}=\mathrm{V} / \mathrm{R}_{2}=220 / 15=14.67 \mathrm{~A}$
The current drawn by both, $\mathrm{I}_{1}+\mathrm{I}_{2}=26.89 \mathrm{~A}$ or, rounded, 26.9 A

Example 2: A parallel circuit has three bulbs of resistance 14 and $26 \Omega$ (when hot). When the bulbs are connected to a battery, a current of 0.45 A flows through the $26 \Omega$ resistor.
(a) Would you expect the voltage across each of the bulbs to be different? Or not? Explain your reasoning.
(b) What is the voltage of the battery?
(c) What is the voltage difference across each of the bulbs?
(d) Calculate the current through the $16 \Omega$ bulb.
(e) What is the total current that must be produced by the battery?

## Solution:

$R_{1}=14 \Omega ; R_{2}=26 \Omega ; I_{2}=0.45 \mathrm{~A}$
$\mathrm{V}=$ ? $; \mathrm{I}_{1}=$ ? Total current = ?
(a) Since the bulbs are in parallel, each one is connected as if it is in an independent circuit with the battery. Therefore every bulb feels the same voltage, in this case, full voltage of the battery.

(b) To calculate the battery voltage, we need the current through one of
the bulbs and its resistance. Since we know $I_{2}$ and $R_{2}$, Battery voltage $\mathrm{V}=\mathrm{I}_{2} \times \mathrm{R}_{2}=0.45 \times 26=11.7 \mathrm{~V}$
(c) 11.7 V (for the reason stated in (a))
(d) $I_{1}=V / R_{1}=11.7 / 14=0.84 \mathrm{~A}$
(e) The total current will be $\mathrm{I}_{1}+\mathrm{I}_{2}=0.84+0.45=1.29 \mathrm{~A}$

Example 3: Rajni finds four resistors in a drawer: $20 \Omega, 12 \Omega, 15 \Omega$ and $10 \Omega$.
(a) By combining them, what are two the largest resistances she can make? Draw a picture of how she had to connect them.
(b) What are the two smallest resistances she can obtain? Draw a picture.

## Solution:

(a) To produce a resistance larger than that of the original resistors she must connect them in series. The largest value would be with all four resistors in series:

$$
R_{1}=20+12+15+10=57 \Omega
$$

The next largest will be if the three largest ones are connected in series:

$$
R_{2}=20+15+12=47 \Omega
$$

(b) To produce resistors smaller than the original she must connect them in parallel.

| The smallest value is when all are connected in <br> parallel: | To produce the next larger resistor, one must <br> use the three smallest resistors in parallel: |
| :--- | :--- |
| $\frac{1}{R_{3}}=\frac{1}{20}+\frac{1}{12}+\frac{1}{15}+\frac{1}{10}$ | $\frac{1}{R_{4}}=\frac{1}{12}+\frac{1}{15}+\frac{1}{10}$ |
| $\frac{1}{R_{3}}=\frac{3+5+4+6}{60}$ | $\frac{1}{R_{4}}=\frac{5+4+6}{60}$ |
| $\frac{1}{R_{3}}=\frac{18}{60}$ | $\frac{1}{R_{4}}=\frac{15}{60}$ |
| $R_{3}=\frac{60}{18}=3.33 \Omega$ | $R_{4}=\frac{60}{15}=4 \Omega$ |
| The smallest resistor has an equivalent resis- |  |
| tance of $3.33 \Omega$ (20, 12,15 and 10 in parallel). | The next smaller resistor has an equivalent re- |
| sistance of $4 \Omega .(12,15$ and 10 in parallel) |  |

## Where does the equation for parallel resistance come from?

The expression arises from applying the following concepts which we have seen before:

- The total current provided by the battery is the sum of the currents in the individual parallel branches.
- The voltage difference across each of the resistors in a parallel circuit is the same.
- Ohm's law: $\mathrm{V}=\mathrm{IR}$.

Let's look at a three-bulb parallel circuit. We know already that the total current I put out by the battery is
$I=I_{1}+I_{2}+I_{3}$.
Since all three resistors see the same
 voltage V , we can write the currents $\mathrm{I}_{1}, \mathrm{I}_{2}$, and $\mathrm{I}_{3}$ in terms of the voltage and the resistance using Ohm's law,
Substituting for $\mathrm{I}_{1}, \mathrm{I}_{2}$, and $\mathrm{I}_{3}$ in the previous equation for I , we get $I=\frac{V}{R_{1}}+\frac{V}{R_{2}}+\frac{V}{R_{3}}$
$I_{1}=\frac{V}{R_{1}} ; \quad I_{2}=\frac{V}{R_{2}} ; \quad I_{3}=\frac{V}{R_{3}}$

The current I on the left-hand side represents the total current provided by the battery. We could, instead, build another circuit with the same battery of voltage $V$ and get the same current I in the circuit if we had an appropriate resistor which we shall call $R_{p}$. Ohm's Law would tell us that
$I=\frac{V}{R_{P}}$
Since the currents in the above two equations are the same, the right-hand sides must be equal:
$\frac{V}{R_{P}}=\frac{V}{R_{1}}+\frac{V}{R_{2}}+\frac{V}{R_{3}}$
Dividing out the common factor $V$ gives:

$$
\begin{aligned}
& \frac{1}{V}\left(\frac{V}{R_{P}}\right)=\frac{1}{V}\left(\frac{V}{R_{1}}+\frac{V}{R_{2}}+\frac{V}{R_{3}}\right) \\
& \frac{1}{\mathrm{~K}}\left(\frac{\mathbb{K}}{R_{P}}\right)=\frac{1}{\nmid X}\left(\frac{\not X}{R_{1}}+\frac{\not X}{R_{2}}+\frac{\not X}{R_{3}}\right) \\
& \frac{1}{R_{P}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}+\frac{1}{R_{3}}
\end{aligned}
$$

$R_{P}$ is the total or equivalent resistance. This equivalent resistance $R_{P}$ feels the same voltage difference across it as $R_{1}, R_{2}$, or $R_{3}$ do, and it draws the same current from the same battery as the group. While this formula was derived for three resistors, a similar formula holds for any number of resistors.

## Reading Page: Power and Energy

## Electrical Power

In an electrical circuit, the electrical power consumed by a device such as a resistor is given by
Power $=$ Voltage $\times$ Current
Or $\mathrm{P}=\mathrm{VI}$
(acronym: Power = Very Important)
Example 1: A refrigerator bulb uses 40 W of power when it is hooked up to a 110 V electrical line. (a) How much current does it draw? (b) What is the bulb's resistance?

## Solution:

(a) $\mathrm{P}=40 \mathrm{~W} ; \mathrm{V}=110 \mathrm{~V}$;

Because $P=V I$, we can rearrange the equation to give us $I: I=P / V$
Therefore $\mathrm{I}=40 / 110=0.363 \mathrm{~A}$.
A current of 0.363 A flows through the bulb.
(b) Since $V=I R$,
we can rearrange the equation to give us

$$
\begin{aligned}
R & =\frac{V}{I} \\
& =\frac{110}{0.363}=303 \Omega
\end{aligned}
$$

The resistance or the bulb is $303 \Omega$.

Example 2: Anita's kitchen has four light bulbs that come on when she flicks the switch. Each light bulb uses a power of 75 W . How much power do the light bulbs consume every time she turns the lights on?

## Solution:

Power adds up -- if you have several devices, you add the power used by each device to get the total power consumed. Since each bulb consumes 75 W , the total amount is $4 \times 75=300 \mathrm{~W}$.

Example 3: Anita decides to save energy and replaces two of the bulbs in Example 2 with fluorescent bulbs that consume only 15 W of electrical power but give off the same amount of light. How much power do the four light bulbs consume now?

## Solution:

The power used now is $75+75+15+15=180 \mathrm{~W}$.

## The Connection between Power and Energy transferred

Power is defined as the amount of energy used per unit time (energy used / sec). This is a general definition for power.

For example, a "powerful bulb" makes us think of a very bright bulb -- namely, one that produces a lot of light. Compare this phrase to "powerful person," which might make you imagine a strong weightlifter, who can pick up a heavy box and carry it upstairs in ten seconds, because his/her muscles can produce a lot of energy in those ten seconds. A weakling, in contrast, may produce the same total amount of muscular energy, but over several minutes -- perhaps s/he needs to unpack the box and carry things up piece by piece. The same amount of energy was expended by both people, but the weakling took longer than the strong person, and is therefore not as "powerful." So when we speak of power, there are two factors: the amount of energy, and the amount of time.

Since Power = Energy / time interval, the amount of energy consumed is
Energy $=$ Power $x$ time interval or
$E=P \Delta t$ (where E = energy, $\mathrm{P}=$ power and $\Delta \mathrm{t}=$ time interval)
Energy is measured in units of joules, or J; 1 joule $=1$ watt $\times 1 \mathrm{sec}$. Conversely, watts $=$ joules $/ \mathrm{sec}$.

## Calculating Electrical Energy

The electric company usually measures energy consumption in watt-hours or in thousands of watt hours (kilowatt hours, KWH).

Example 4. Gina uses a 1200 W hair dryer for half an hour. How much energy has she used in (a) watt-hours (b) kilowatt-hours?

## Solution:

(a) Power $\mathrm{P}=1200 \mathrm{~W}$; Time $\Delta \mathrm{t}=30 \mathrm{~min}=0.5$ hours

Energy $=P \Delta t=1200 \mathrm{~W} \times 0.5 \mathrm{~h}=600$ watt-hours
(b) Since we want energy in kilowatt-hours, we should convert power to kilowatts (KW). Since 1000
$\mathrm{W}=1 \mathrm{KW}, \mathrm{P}=1200 \mathrm{~W}=1.2 \mathrm{KW} ; \Delta \mathrm{t}=0.5 \mathrm{hr}$
$E=P \Delta t=1.2 \mathrm{KW} x 0.5 \mathrm{~h}=0.6 \mathrm{KWH}$.
Our electric bills calculate energy consumption. Since both power and time figure into the calculation of energy, reducing both factors lowers our electric bill. A 60 W light bulb uses more power than a 40 W light bulb; alternatively, one could say that the 60 W light bulb uses more energy per second than a 40 W bulb. The more power an appliance consumes, and the longer we have it on, the more energy it uses -- and the bigger the electric bill. Our bills would be lower if we kept the lights on for fewer hours in the day; or used less powerful bulbs - 60 W instead of 100 W ; better yet, fluorescent bulbs, which consume about $1 / 4$ the power but produce the same amount of light.

A serious drain on electrical energy is caused by having a large number of electrical appliances on all the time. Many devices, such as computers, TVs and stereo equipment, to name a few, go into a "sleep" mode where they stay on but consume much less energy. While this seems like a great idea, the rapid increase in the number of such devices has increased the fraction of total energy consumed in such low power modes. The industry is currently aiming to have devices use 1 W or less in their low power modes.

## Reading Page: AC and DC

## AC and DC

The symbol for AC:
When a battery is connected to a resistor, the electrons drift steadily in one direction -- from the negative side of the resistor to the positive side. The + and - sides of the resistor do not change over time. This is called direct current, or DC.


Electric power plants, in contrast, produce alternating current, or AC. An AC voltage switches the + and - sides of the resistor back and forth many times a second. As a result, electrons start out in one direction, then turn around and go backwards, then forwards again. Since this reversal happens very quickly, the electrons may just jiggle back and forth and not drift too far from their usual thermal paths. In the USA and Canada, the full cycle of 0 to +V to 0 to -V to 0 occurs every $1 / 60$ th of a second. AC is usually designated by a wavy symbol, as shown in the picture.

## How is energy transferred by AC?

If the electrons do not go anywhere, one might ask, how can one have current in an AC circuit, and why is energy transferred? While the electrons do not drift far, they still jiggle back and forth more than they did when there was no voltage difference across the resistor. It is the motion of electrons, even if it is microscopic, that produces a current and the transfer of energy to neighboring vibrating electrons throughout the resistor. Thus the small but persistent vibration with a cycle of $1 / 60$ th of a second transfers energy to the resistor or other circuit element.

## Resources:

Movie on inspection of High Voltage lines:

[^0]
[^0]:    http://www.wxpnews.com/706MVW/070424-Cable-Inspector

