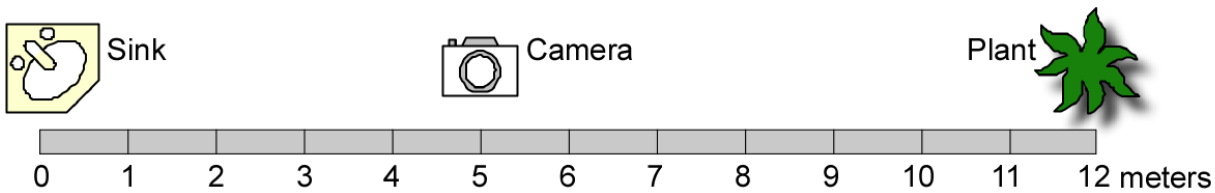
# *Reading Page:* What is Position?

**What is position?**

When you ask, “Where is the dictionary?” you are asking for the *position* of the object. In everyday language we might say, “it is on the table” or “it is next to the Harry Potter book on the bookshelf.” In either case, we define the location of the dictionary relative to another object whose position we already know. Or we might say, (after checking the GPS) “it is at latitude 38° 58’ 22”N and longitude 92° 22’ 43” W,” in which case we are defining the dictionary’s position relative to a specifically defined zero of latitude and longitude.

When we define position in the lab, it is easiest to define it in terms of how far it is from a specified “zero” mark:



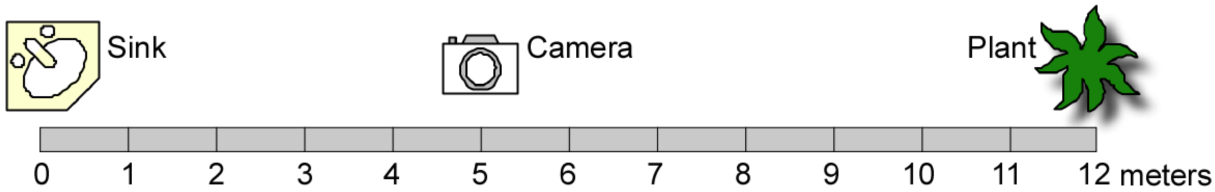

In the picture above, we could say that the camera is 5 meters (m) to the right of the sink. The plant is 12 m to the right of the sink. These statements imply that if the zero mark of a ruler was at the sink, the position of the camera is 5 m to the right and the position of the plant is 12 m to the right of the zero mark. The zero of the measuring tape or ruler is at an arbitrary but known position (in this case, the sink). This is the methodology we will use to define the position of objects while we study motion.

The position of an object can <u>only</u> be defined relative to another object, or mark, or specified zero position.

# *Reading Page:* Distance and Change in Position

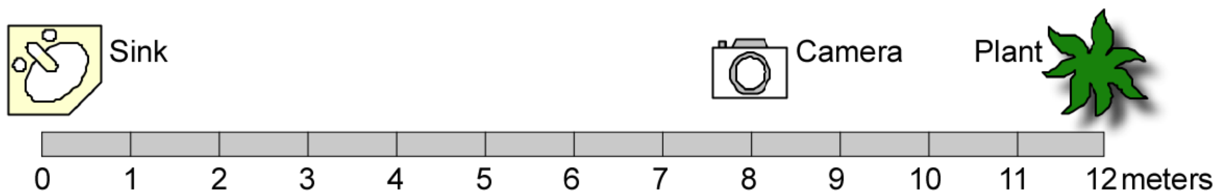
**Position?**

Position is defined in terms of how far an object is from another object, or a specified "zero" mark: The position of an object can only be defined relative to another object, or mark, or specified zero position.




What happens when an object moves? We use the same terminology.

**Example 1**: In the picture below the camera has moved from 5 m to 8 m from the sink. How much did its position change?



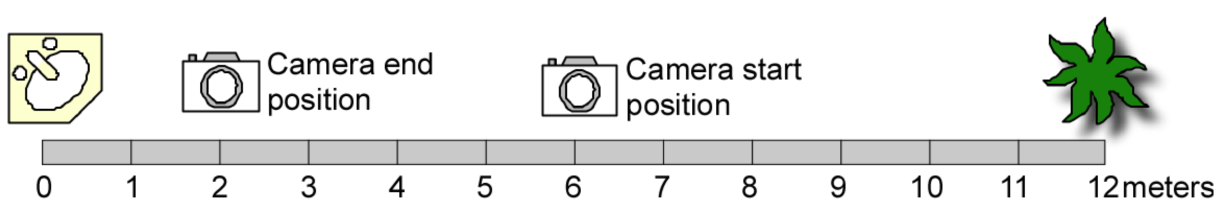

To keep things straight, we define the starting and ending positions of the objects: the starting or *initial position* is defined as $x_i = 5$ m. The ending or *final position* is defined as $x_f = 8$ m. The change in the camera's position is $\Delta x$ = final position – initial position

$\Delta x = x_f - x_i$
$\Delta x = 8 - 5 = 3$
$\Delta x = 3$ m

Unit check: if all our starting values are in consistent units, the units will be consistent when we reach the end of the calculation. Here $x_f$ *and* $x_i$ were in meters (m). Therefore $\Delta x$ will also be in m.

The *change in position* is also called *displacement*.

**Example 2**: The camera is initially at 6 m and moves so that it is 2 m from the sink. How much did its position change?




Here $x_i = 6$ m, and $x_f = 2$ m
$\Delta x = 2 - 6 = -4$
$\Delta x = -4$ m

A positive $\Delta x$ indicates that the object moves away from the zero mark. A negative $\Delta x$ indicates that the object moves toward the zero mark. .

**What is distance?**

Distance is the total length traveled. The distance along a straight line is the difference between the position readings. However, distance is defined as a positive quantity. Whether the object moves to the left or the right, the distance is always positive, while the change in position can be positive or negative. The distance does not contain information about the direction of motion, while the change in position does.

Mathematically, we write distance as the *magnitude* (also called the *amount* or *absolute value*) of the change in position.   This is indicated by the standard mathematical symbol $|\Delta x| = |x_f - x_i|$

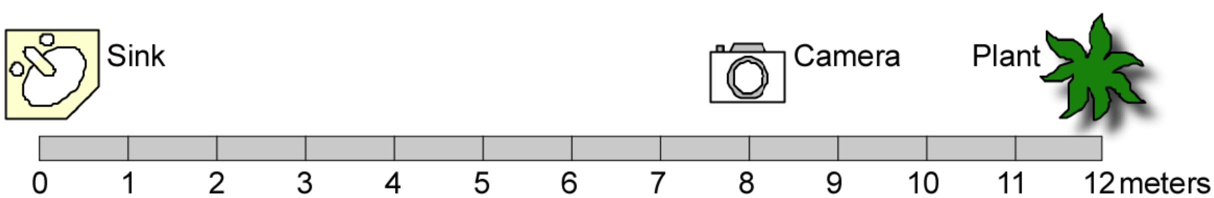
In example 1, the distance the camera moves is $|\Delta x| = |8 - 5| = 3$ ;  $|\Delta x| = 3$ m.
In example 2, the distance the camera moves is $|\Delta x| = |2 - 6| = |-4| = 4$; $|\Delta x| = 4$ m.

The change in position is the difference between the final and initial position readings.
The distance along a straight line is the magnitude (or amount) of the change in position .

**Example 3:** A bug travels from the camera to the sink and then to the plant.

a) Calculate the total distance traveled.

b) Calculate the total change in position (displacement).




*Solution:*

a) The total distance traveled is $d = |\Delta x_1| + |\Delta x_2|$

$|\Delta x_1| = |x_{sink} - x_{camera}| = |0\text{-}8| = 8$
$|\Delta x_1| = 8$ m

$|\Delta x_2| = |x_{plant} - x_{sink}| = |12\text{-}0| = 12$
$|\Delta x_2| = 12$ m

$d = |\Delta x_1| + |\Delta x_2| = 8 + 12 = 20$
$d = 20$ m                    (intermediate positions *do* matter).

b) The total change in position is $\Delta x = x_f - x_i$

Here the initial position is $x_i = x_{camera} = 8$ m
The final positon is $x_f = x_{plant} = 12$ m
$\Delta x = x_{plant} - x_{camera} = 12 - 8 = 4$
$\Delta x = 4$ m                    (intermediate positions *do not* matter)

To summarize, If you go from A to B, then B to C, and back again to A, the total distance is the sum of the three distances: AB + BC + CA (just as you would read in the odometer of a car).

A                    B                    C

**Example 4:** In Paris, the zero mark is located at the Eiffel Tower. The airport is 20 km to the west of the Tower. The bus station in your neighborhood is 35 km to the east of the Tower.

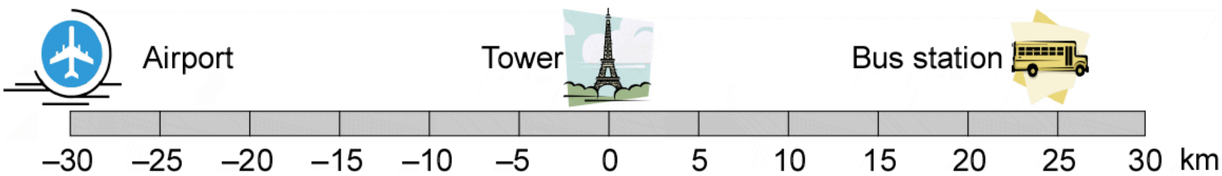
a) Make a schematic diagram of this situation.

b) Calculate the distance you travel when you take a bus from the bus station to the airport

c) Calculate your change in position.

*Solution:*

a) Schematic diagram:




b) The distance between the bus station and the airport is
$d = |x_f - x_i| = |(-30) - 25| = |-65|$
$d = 65$ km

c) The change in position between the bus station and the airport is
$\Delta x = x_f - x_i = (-30) - 25 = -65$
$\Delta x = -65$ km

**Summary of symbols used:**

Here are the symbols we have used and will use:

| | |
|---|---|
| $x_i$ for initial or starting position<br>$x_f$ for final or ending position<br>$\Delta x$ for displacement (change in position)<br>$\lvert\Delta x\rvert$ or $d$ for distance | $t_i$ for initial or starting clock reading<br>$t_f$ for final or ending clock reading<br>$\Delta t$ for the time interval |

# *Reading Page:* Unit Conversion

Physical quantities such as time or distance are often expressed in a variety of different units. For example, distance may be measured in miles, cm or meters. Time may be measured in seconds, hours or days. A simple method of converting units is shown below.

**Concepts involved in unit conversion:**

When we write a conversion as 1 inch = 2.54 cm, we are writing an equation. We can therefore do the following:

$1 inch = 2.54 cm$

Divide both sides by 2.54 cm

$$\frac{1 inch}{2.54 cm} = \frac{2.54 cm}{2.54 cm}$$

Cancel common factors

$$\frac{1 inch}{2.54 cm} = \frac{\cancel{2.54 cm}}{\cancel{2.54 cm}}$$

$$\frac{1 inch}{2.54 cm} = 1$$

**AND we can also do the following:**

$1 inch = 2.54 cm$

Divide both sides by 1 inch

$$\frac{1 inch}{1 inch} = \frac{2.54 cm}{1 inch}$$

Cancel common factors

$$\frac{\cancel{1 inch}}{\cancel{1 inch}} = \frac{2.54 cm}{1 inch}$$

$$\frac{2.54 cm}{1 inch} = 1$$

Since we can multiply any value by 1 and not change it, let's do the following (see box):

$7 inch = 7 inch \text{ x } 1$

$$= 7 inch \text{ x } \frac{2.54 cm}{1 inch}$$

Canceling the common units,

$$= 7 \cancel{inch} \text{ x } \frac{2.54 cm}{1 \cancel{inch}} \text{, gives}$$

$$= 7 \text{ x } 2.54 cm = 17.78 cm$$

In this example we chose the appropriate substitution for "1" -- namely, the one that allowed us to cancel the "inch" unit in the numerator with the "inch" unit in the conversion factor, leaving us with a "cm" unit in the numerator -- and in the process, converting inches to cm.

By the way, if we chose the other substitution for "1", we would not cancel units, so it is not useful for unit conversion.

We often skip the step of rearranging to get a "1" by just writing the conversion directly in its fractional form, so it looks like a "vertical equation" (see Example 1). This method can also be used to convert several units together.

To summarize, here's the process:

- Write the number *and* units you want to convert.
- Write the unit conversion (e.g., 1 inch = 2.54 cm) vertically, so that the unit you want to change gets cancelled out. If you want to convert cm in the numerator to inches, put 1 inch in the numerator and 2.54 cm in the denominator.
- Cancel all common units.
- Multiply the numbers. Preserve all the units that have not been cancelled.

**Example 1**: Convert 5 cm to inches.

Solution: Since 1 inch = 2.54 cm,

$$5 cm = 5 cm \times \frac{1 inch}{2.54 cm}$$

$$= 5 \cancel{cm} \times \frac{1 inch}{2.54 \cancel{cm}} = 1.97 inch$$

**Example 2**: Convert an area of 4 $m^2$ to units of $cm^2$.

Solution: The process works for multiple units too. The conversion factor is written twice so we can cancel the square of cm:

$$4m^2 = 4m^2 \times \frac{100cm}{1m} \times \frac{100cm}{1m}$$
$$= 4\cancel{m^2} \times \frac{100cm}{1\cancel{m}} \times \frac{100cm}{1\cancel{m}} = 4000cm^2$$

Conversion: 100cm = 1m

Note: Instead of the multiply (x) sign, one frequently writes a vertical line (|) Instead (as a short cut)

**Example 3**. Here's a long one: Convert one year into seconds.

Solution:

$$1year = 1\,\cancel{year} \times \frac{365\,\cancel{days}}{1\,\cancel{year}} \times \frac{24\,\cancel{hours}}{1\,\cancel{day}} \times \frac{60\,\cancel{\min}}{1\,\cancel{hour}} \times \frac{60\sec}{1\,\cancel{\min}}$$
$$1year = 31{,}536{,}000\sec$$

which can also be written as:

$$1year = 1\,\cancel{year}\left|\frac{365\,\cancel{days}}{1\,\cancel{year}}\right|\frac{24\,\cancel{hours}}{1\,\cancel{day}}\left|\frac{60\,\cancel{\min}}{1\,\cancel{hour}}\right|\frac{60\sec}{1\,\cancel{\min}}$$
$$1year = 31{,}536{,}000\sec$$

**Example 4**. Convert 14 km/hour into m/sec.

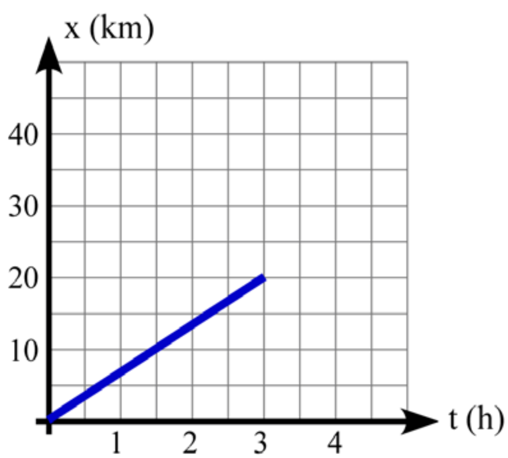
Solution: From the conversion tables in the appendix, 1 km = 1000m and 1 hr = 3600s

$$14km/hr = 14\frac{\cancel{km}}{\cancel{hr}}\left|\frac{1000m}{1\cancel{km}}\right|\frac{1\cancel{hr}}{3600s}$$
$$= \frac{14 \bullet 1000m}{3600s} = 3.89m/s$$

**Example 5**. In an equation such as Ohm's Law, voltage = current times resistance, or V= IR also implies that the units multiply: thus volts = amps x ohms. For example, if I = 3 amps and R = 4 ohms,

V = IR = 3 amps x 12 ohms = 36 amps x ohms = 36 volts.

**Example 6.** In the graph, calculate the slope and convert it to m/sec.



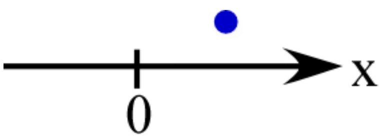
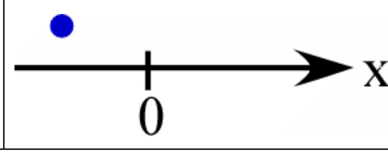

$$\text{Slope, } v = \frac{rise}{run} = \frac{(20-0)km}{(3-0)hr}$$
$$= \frac{20km}{3hr} = 6.67\frac{km}{hr}$$

Conversion to m/s

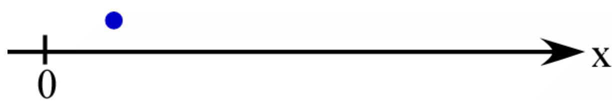
$$6.67\frac{km}{hr} = 6.67\frac{km}{hr}\left|\frac{1000m}{1km}\right|\frac{1hr}{60\min}\left|\frac{1\min}{60\sec}\right.$$
$$= 6.67\frac{\cancel{km}}{\cancel{hr}}\left|\frac{1000m}{1\,\cancel{km}}\right|\frac{1\,\cancel{hr}}{60\,\cancel{\min}}\left|\frac{1\,\cancel{\min}}{60\sec}\right.$$
$$= \frac{6.67 \text{ x } 1000m}{60 \text{ x } 60\text{sec}} = 1.85\frac{m}{s}$$

# *Reading Page:* Motion Diagrams

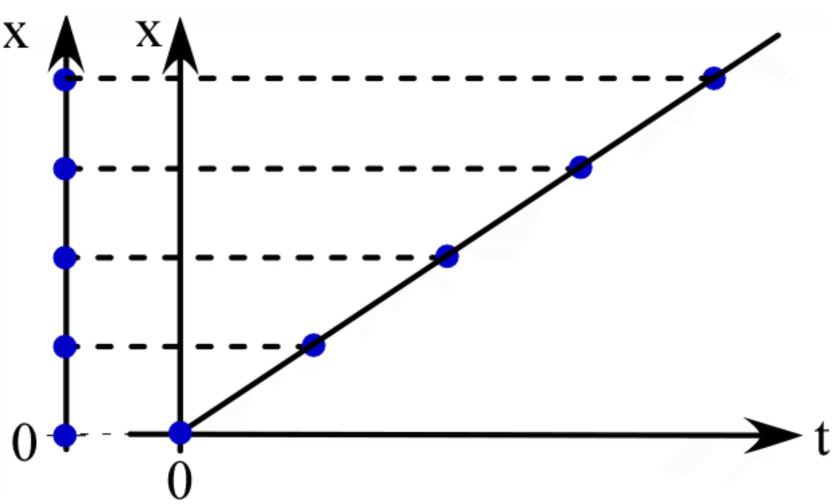
A motion diagram is like a composite photo that shows the position of an object at several equally spaced time intervals, like a stroboscopic photograph. We model the position of the object with a small dot or point with reference to the origin of a coordinate axis. The origin may be arbitrary.

| Position to right of origin, x is positive   | Position to left of origin, x is negative   |
|---|---|

**Examples of motion diagrams:**

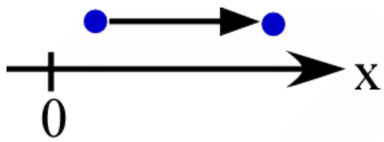
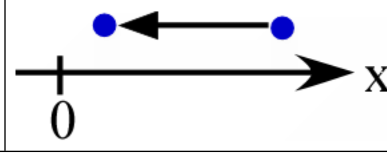
| A stationary ball on the ground   | An object that is at rest occupies only a single position in a motion diagram   |
|---|---|
| A skateboarder rolling down a sidewalk   | An object that moves with constant speed produces images that are equally spaced (since it travels equal distances in equal time intervals). Its motion diagram is a series of equally spaced dots.   |

How is a motion diagram related to the position versus time graph? See below. The motion diagram is like an x-t graph collapsed onto one axis.

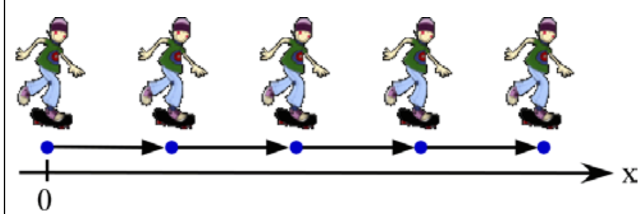
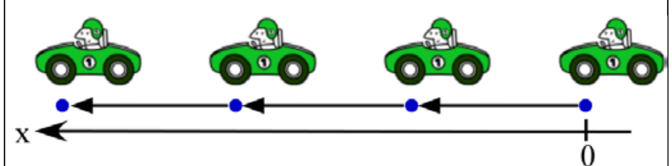


**Motion diagrams for objects moving with constant speed**

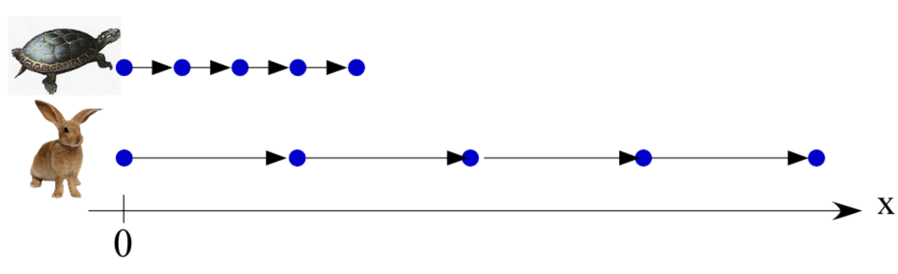
In addition to the position, we can also represent the speed of the object on the motion diagram. An arrow is associated with each pair of position dots; the length of the arrow shows the distance traveled in one time interval. In other words, the arrow's length represents the amount of speed (longer arrow, larger speed) and the direction of the arrow indicates the direction of motion of the object. When the direction and the speed are specified, it is called velocity, indicated by *v*.

| | |
|---|---|
| Direction of motion is to the right, in the positive direction, *v* is positive   | Direction of motion is to the left, in the negative direction, *v* is negative   |

**Examples:**

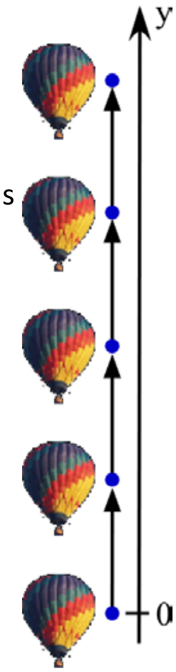
| | |
|---|---|
| Example 1: A skateboarder is rolling down the sidewalk with a constant speed. The distance between the points is the same, and the length of the arrows is the same, indicating that the object moves with constant speed. The arrows are oriented in the direction of motion of the skater. |   |
| Example 2: A car is moving to the left at a constant speed. The distance between the points is the same, and the length of the arrows is the same, indicating that the object moves with constant speed. The arrows point along the direction of motion of the car. The positive direction of motion can be chosen toward the right or to the left. Here the positive direction is chosen to be toward the left. |   |

Example 3: A tortoise and a rabbit have a race. Each runs with a constant speed but the rabbit runs 4 times faster than the tortoise. Their motion is represented on the same motion diagram.




The dots for the position of the rabbit and for the tortoise are drawn for the same time intervals. The arrow the rabbit's speed is four times longer than that for the tortoise's speed - which implies that the rabbit runs four times faster. Even though the dots for the two animals line up at one position along the x-axis, it does not mean that the two objects have the same speed or at the same position at the same time. It means that the rabbit reached that position after one time interval, while the tortoise reached the same position after 4 time intervals have passed.

Example 4: A balloon is ascending (moving up) at constant speed.
The distance between the points is the same (position points are equally spaced), and the length of each arrow is the same, indicating an object that moves with constant speed. The direction of the arrow indicates the direction of motion of the balloon.




Note: the axis is named the "y-axis" because vertical positions are usually designated as y, not x.

**Summary of Motion Diagrams**

A motion diagram is a series of snapshots of the motion of the object, taken at regular time intervals.

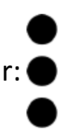
A motion diagram depicts the motion of the object over a chosen time period

A motion diagram is qualitative – specific numbers are not necessary. For this reason, there is no set interval (e.g., 1 sec or 2 sec) between dots.

A motion diagram should specify the direction of +x

A motion diagram should specify the starting point (t=0)

If the object stands still, indicate it with a single dot: 

If the object stands still for several clock ticks, indicate it with several dots one above another: 

If the object stands still for a long time, draw a circle around the dot. 

If an object moves upward, the motion diagram is drawn vertically

If an object moves horizontally to the left, the motion diagram is drawn to show motion to the left.

If the object moves on a slope, the motion diagram is drawn on a slope.

In a motion diagram you need at least two time intervals to show constant speed

# *Reading Page*: The Speed-Distance-Time Relation

So far we have been examining situations where the speed stays the same over the time interval of our measurement. This situation can be represented in several ways: via a motion diagram, a graph, or in words. Mathematically, we can relate distance, time and speed by

$$Speed = \frac{Change\ in\ position}{Time\ \ taken}$$

Where:

$$or,\ v = \frac{\Delta x}{\Delta t}$$

$v$ = speed (the amount of velocity)

$|\Delta x| = |x_f - x_i|$, the distance traveled between final ($x_f$) and initial ($x_i$) positions, often written as just $\Delta x$

$\Delta t = t_f - t_i$ ; the time taken is the difference between the final and initial clock readings, $t_f$ and $t_i$, respectively; also called time for travel.

**Units:**

$\Delta x$ is in units of length;

$\Delta t$ is in units of time;

$v$ is in units of $\frac{\text{length}}{\text{time}}$

Several different units are used for speed. Speed is always measured in units of length/ time. Metric units: meters/sec or cm/sec.

| [distance] | [time] | [speed] |
|---|---|---|
| miles | hours | miles/hour |
| cm | sec | cm/sec |
| mm | day | mm/day |
| meters | sec | meters/sec |

***Problem-Solving Strategy:***

- Read the problem carefully
- Identify the concept addressed
- Draw a diagram (needed frequently)
- Choose suitable units: cm-gram-sec or SI (meter-kg-sec) are convenient
- Identify and list the data given and the questions asked
- Convert data to chosen units
- Identify the concept involved and formulae needed
- Perform algebraic manipulations, if necessary
- Calculate and verify: is the answer reasonable? Are signs and units correct?
- Write a sentence stating the answer, including units.

*Some of these steps may be combined or interchanged in simple problems*

*Alternative method: GUPPieS[2]*

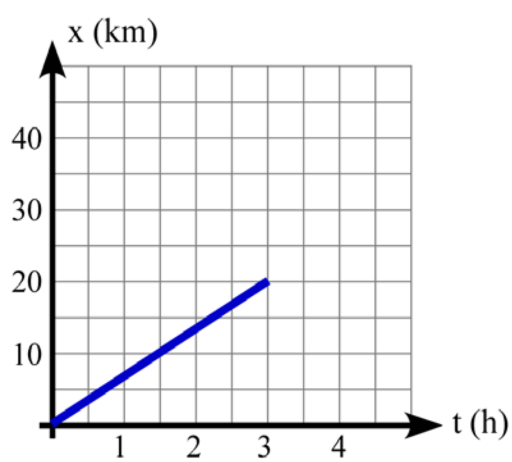
**Example 1:** Ari runs a distance of 20 km in 3 hours at constant speed. How fast does he travel? Draw a graph and a motion diagram of his motion.

*Solution:*
Distance $\Delta$x= 15 km;
Time $\Delta t$ = 3 hours (h)
$v$ = ?

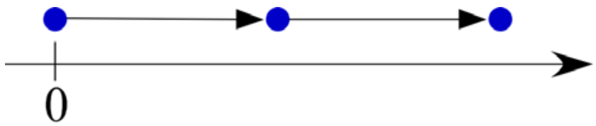
*Concept:* Since Ari runs at a constant speed, we can use speed v = $\Delta$x /$\Delta t$ to do this problem.

$$v = \frac{|\Delta x|}{\Delta t} = \frac{20}{3} = 6.67$$
$$v = 6.67\,km\ /\ hr$$

Ari runs at 6.67 km/h.



The motion diagram for Ari's motion:




**Example 2:** A centipede travels at a speed of 2 mm/sec. How far does it get in six minutes?

*Solution:* When we make calculations it is a good idea to choose the units we will use and stick by them. This example uses mm for distance and sec for time (2 mm/sec) when the speed is defined. Later in the problem time is defined in minutes. Let's make a choice of units first:

*Choose units:* Distances will be defined in mm
Time will be defined in seconds.

*Convert to chosen units:*
The speed, in mm and sec, is consistent with our choice above. We have to convert the total time of travel, $\Delta t = 6$ min, into seconds:

$$\Delta t = 6\,\text{min} = 6\,\text{min}\left|\frac{60\,\text{sec}}{1\,\text{min}}\right. = 6 \times 60\,\text{sec} = 360\,\text{sec}$$

It is important to identify variables, and list the data and questions in the problem. This process might seem like a waste of time for simple problems, but turns out to be a life-saver when you get to more complex problems. It keeps track of the data that you will need for the "Calculate" step. Get used to this great habit!

*List data and questions:*
Speed v = 2 mm/sec
Time $\Delta t$ = 360 sec
Distance $\Delta x$ = ?

*Identify concept and formula and calculate:* The centipede moves with a constant speed.

$$v = \frac{\Delta x}{\Delta t}$$

$$2 = \frac{\Delta x}{360}$$

$$= 2\left[\frac{mm}{s}\right] \cdot 360[s] = 720mm$$

The centipede travels 720 mm (0.72 m)

Note: We can include units within a calculation (as shown above), or to put it in at the end. If you choose units when you begin the problem and make sure that all quantities have consistent units, the units will come out correct at the end of the problem.

**Example 3.** My dog Toto runs at a speed of 15 km/hr. If she runs a distance of 3250 m,

a) How long will she take?

b) Draw a motion diagram

c) Draw an x-t graph, and check the slope of the graph with the given value of speed.

*Solution:*

*Choose units:* Distances in km, time in hours.

*Convert to chosen units, list givens and unknowns:*

Speed v = 15 km/hr; distance = ?

Distance: $\Delta x = 3250m = 3250m\left|\frac{1km}{1000m}\right. = \frac{3250km}{1000} = 3.25km$

*Identify concept and formula and calculate:*

a)Toto runs with a constant speed. Therefore the time she takes is,

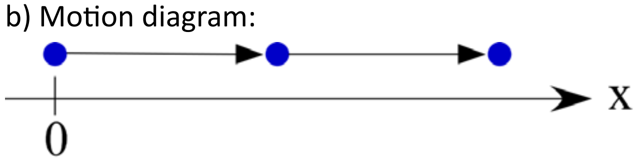
$$v = \frac{\Delta x}{\Delta t}$$

$$15\left[\frac{km}{hr}\right] = \frac{3.25km}{\Delta t}$$

$$\Delta t = \frac{3.25km}{15\left[\frac{km}{hr}\right]} = 0.21hr$$

Toto runs for 0.21 hours.

b) Motion diagram:



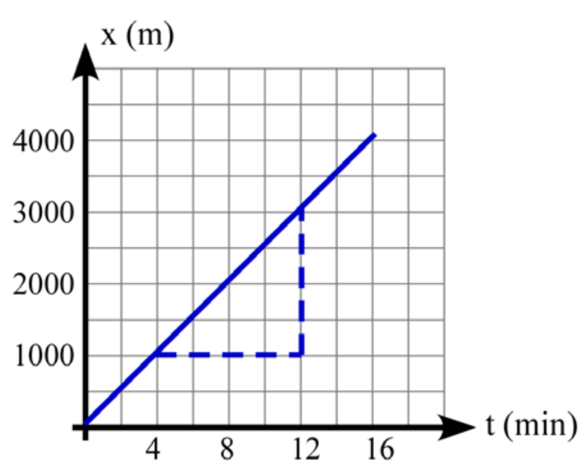

c) x vs t graph.

Checking speed using the slope:

This value of 15 km/hr is equal to the given value of Toto's speed.

$$\text{v} = \frac{3000 - 1000}{12 - 4} = 250\ \text{m / min}$$

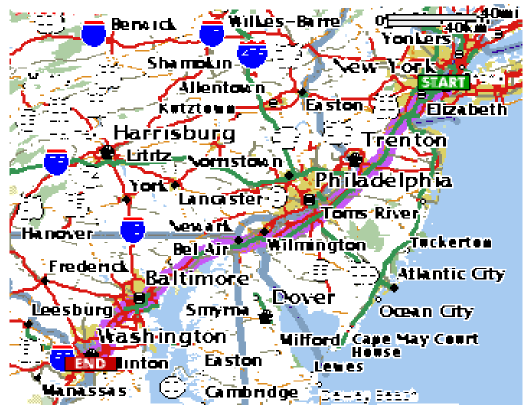
$$250\frac{\text{m}}{\text{min}}\left|\frac{60\ \text{min}}{1\ \text{hour}}\right|\frac{1\ \text{km}}{1000\ \text{m}} = 15\ \text{km / h}$$



# *Reading Page*: Average Speed

**What is Average Speed?**

We frequently need to calculate the average speed at which we travel. For example, trucker Martha is hauling a load from New York to Kansas City, a distance of 1200 miles. She knows that she will travel close to the speed limit most of the time. The speed limits are different in the different states – they range from 55 mph to 70 mph. Moreover, she will be slowed down by traffic, weather, and for rest breaks. What matters most to her is how long it takes to travel that distance. If she knows an average speed from this trip, then she can easily estimate how long a different trip, say, from Kansas City to Toronto, might take.




Average speed is defined as

$$v_{avg} = \frac{\text{total distance}}{\text{total time}} = \frac{|\Delta x_1| + |\Delta x_2| + |\Delta x_3| + \ldots}{\Delta t_1 + \Delta t_2 + \Delta t_3 + \ldots}$$

which, for convenience, we can write as

$$v_{avg} = \frac{d_1 + d_2 + d_3 + \ldots}{\Delta t_1 + \Delta t_2 + \Delta t_3 + \ldots}$$

where $|\Delta x_1| = d_1$, etc.

**Example 1.** Martha drives the busy traffic corridor from New York City to Washington DC. She travels 60 miles in 1 hour and 40 min and, the next 60 miles in 50 min and the last 80 miles in 2 hours.

a) Calculate her average speed.

b) Draw a graph of her position as a function of time

c) Draw a graph of her speed as a function of time

*Solution:*
(a) To calculate average speed we need the total distance and total time; so we first figure out the distances and times for each segment.

*Choose units:* use miles for distance, hours for time, and miles/hr for speed.

*List data and questions:*
Segment 1: Distance $d_1$ = 60 miles in time $\Delta t_1$=1hr 40 min = 1 + 40/60 = 1.667 hr
Segment 2: Distance $d_2$ = 60 miles in time $\Delta t_2$ = 50 min/(60 min/hr) =0.833 hr
Segment 3: Distance $d_3$ = 80 miles in time $\Delta t_3$ = 2 hr
Average speed = ?

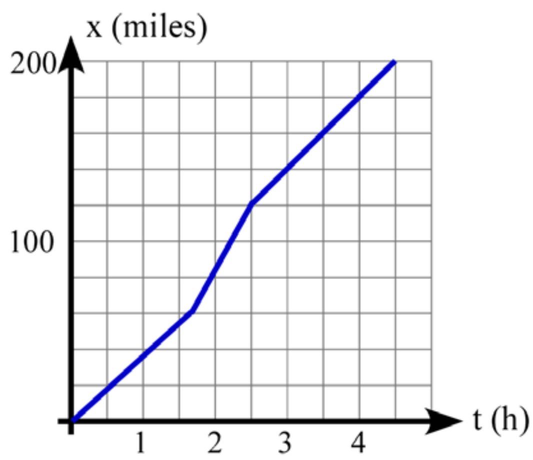
*Identify concepts and formulae and calculate:*

Total distance $d_1 + d_2 + d_3$ = 60 + 60 + 80 = 200 miles
Total time $\Delta t = \Delta t_1 + \Delta t_2 + \Delta t_3$ = 1.667 + 0.833 + 2 = 4.5 hr

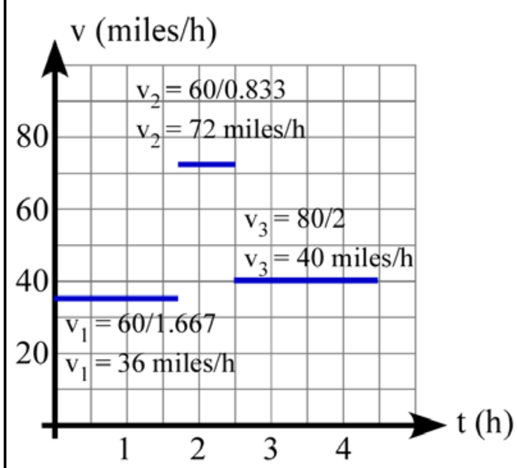
$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{200\text{miles}}{4.5\text{hr}} = 44.4\text{miles / hr}$$

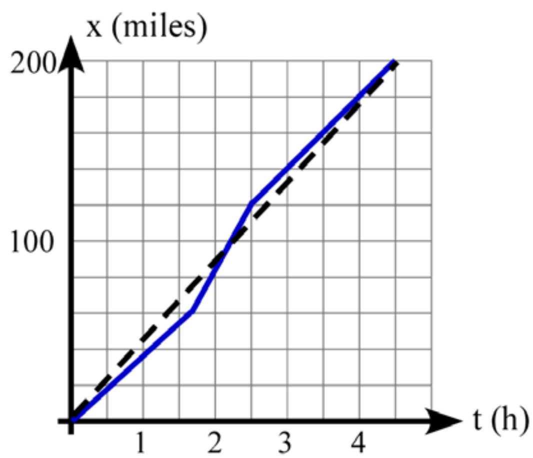
Martha's average speed is 44.4 mph.

(b) Graph of x vs. t




(c) Graph of v vs. t




On the left the dashed line indicates Martha's x-t graph if she had been traveling at a uniform speed. It is drawn by just connecting the first and last points, namely $x_i$ and $x_f$.

Note that the slope of the dashed line is given by total distance/total time - so its slope is the average speed we just calculated.

**The graphical method can be used to calculate the average speed only if the person travels in the same direction** (not if they go forward and then backwards).

Note: Abrupt changes in velocity, as shown in the graphs above, are an idealization and not possible in real life (making them abrupt makes our calculations easier).

**Example 2.** Andy rides his bike on the trail one morning. He travels at 15 km/h for 30 min, and 25 km/h for 45 min. Calculate his average speed.

*Solution:*

*Identify concept:* In this example we are given the speed and time for each of two segments of travel. We will have to first figure out the distance traveled in each of the two segments, then use the formula for average speed.

*Choose units:* km for distance, hours for time, km/h for speed.

Convert data to chosen units:

$$\Delta t_1 = 30\text{ min} = 30\text{min}\left|\frac{1\text{hr}}{60\text{min}}\right. = \frac{30}{60}\text{hr} = 0.5\text{hr}$$

$$\Delta t_2 = 45\text{min} = 45\text{min}\left|\frac{1\text{hr}}{60\text{min}}\right. = \frac{45}{60}\text{hr} = 0.75\text{hr}$$

*List data (in chosen units) and questions:*

Segment 1: speed $v_1$ = 15 km/h for time $\Delta t_1$ = 0.5 h

Segment 2: speed $v_2$ = 25 km/h for time $\Delta t_2$ = 0.75 h

Average speed=?

*Identify concepts and formulae and calculate:*

We first use the speed-distance-time relation d = v$\Delta$t to calculate the distances traveled in each segment:

Segment 1: distance $d_1 = v_1(\Delta t_1) = (15)(0.5) = 7.5$ km

Segment 2: distance $d_2 = v_2(\Delta t_2) = (25)(0.75) = 18.75$ km

We now have all the pieces of information that we need:

Total distance $d = d_1 + d_2$ = 7.5 + 18.75 = 26.25 km

Total time $\Delta t = \Delta t_1 + \Delta t_2$ = 0.5 + 0.75 = 1.25 h

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{26.25 \text{ km}}{1.25 \text{ h}} = 21 \text{ km/h}$$

Andy's average speed is 21 km/h.

Note: Notice that we did not average the two speeds!! If we had, we would have gotten 20 km/h, which is close, but incorrect. Because the two segments represent time periods that are not equivalent, merely averaging the speeds does not weight the segments correctly. (Averaging the speeds is one of the most common errors made in such problems).

**Example 3:** Sherry runs the marathon in her city. She runs the first 8 km at a speed of 11 km/h, the next 13 km at 10 km/h and the last 5 km at 12 km/h. Calculate her average speed.

*Solution:*

In this example we are given speed and distance in three segments of travel. We will have to first find the time taken for each of the three segments, then use the formula for average speed.

*Choose units:* km/h for speed, km for distance and h for time.

*List data and questions:*

Segment 1: distance $d_1$ = 8 km; speed $v_1$ = 11 km/h;
Segment 2: distance $d_2$ = 13 km; speed $v_2$ = 10 km/h;
Segment 3: distance $d_3$ = 5 km; speed $v_3$ = 12 km/h;
Average speed = ?

*Identify concepts and formulae and calculate:*

To calculate the average speed we first calculate the time taken for each of the three segments:

$$\Delta t_1 = \frac{d_1}{v_1} = \frac{8km}{11km/hr} = 0.73hr$$

$$\Delta t_2 = \frac{d_2}{v_2} = \frac{13km}{10km/hr} = 1.30hr$$

$$\Delta t_3 = \frac{d_3}{v_3} = \frac{5km}{12km/hr} = 0.42hr$$

Total distance d = $d_1 + d_2 + d_3$ = 8 + 13 +5 = 26 km

Total time $\Delta t = \Delta t_1 + \Delta t_2 + \Delta t_3$ = 0.73+1.3+0.42 = 2.45 hr

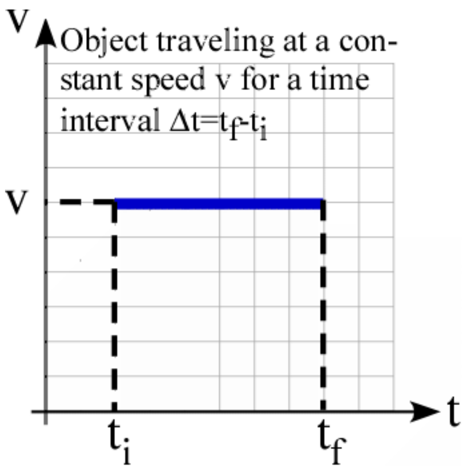
$$\text{Average speed } = \frac{\text{total distance}}{\text{total time}} = \frac{26 \text{ km}}{2.45 \text{ hr}} = 10.6 \text{ km/hr}$$

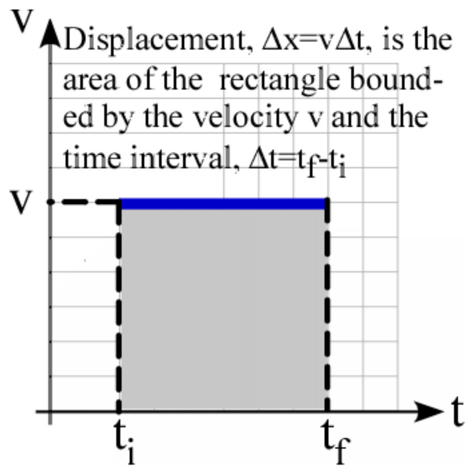
Sherry's average speed is 10.6 km/h.

# *Reading Page* - Calculating Displacement

When the velocity is constant, the displacement (change in position) is given by $\Delta x = v\Delta t$

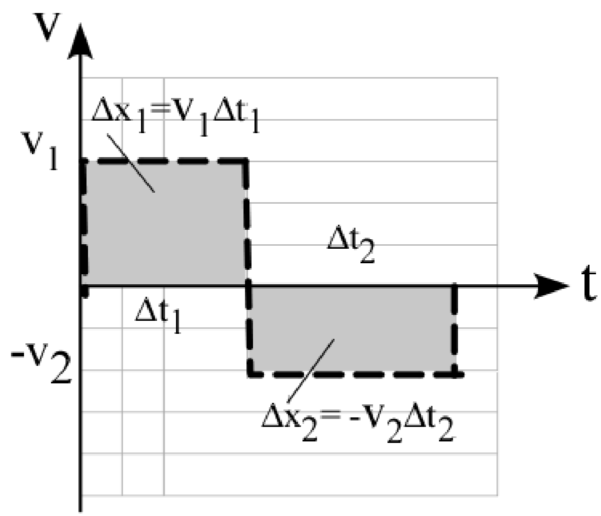
This formula can be understood graphically from the area under the curve. On the left is a graph of an object traveling at a steady velocity v for a time period Δt. The area bounded by the velocity v and the time Δt, gives the rectangle vΔt, which is the displacement, $\Delta x = v\Delta t$.




Therefore, the area under a v-t graph gives the displacement. This holds whether velocity is positive or negative. If the velocity is negative, the displacement is negative, giving us $\Delta x = -v\Delta t$. If we have both a positive velocity $+v_1$ for time $\Delta t_1$ and a negative velocity $-v_2$ for time $\Delta t_2$, the total displacement is $\Delta x_1 + \Delta x_2 = v_1\Delta t_1 + (-v_2\Delta t_2)$.

Graphically, this is how it would look:

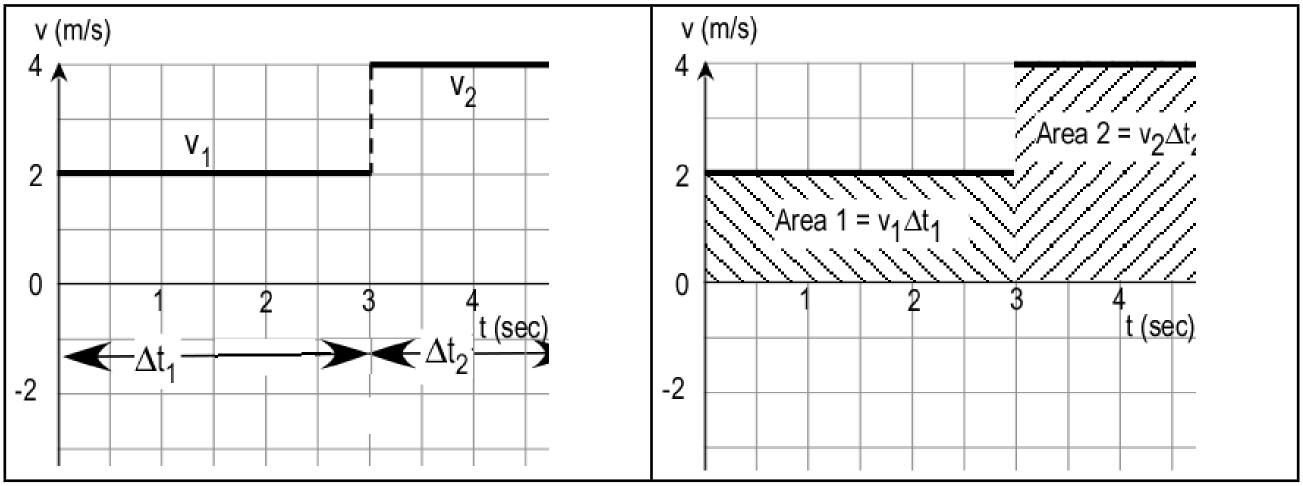



Therefore, the displacement can be calculated mathematically using the formula $\Delta x = v\Delta t$ or by figuring out the area under the curve. These two methods are equivalent.

**Example 1**. A squirrel "walks" with a speed of 2 m/s for 3 seconds, and then scampers at 4 m/s for 2 seconds. Calculate the squirrel's displacement using a graphical method.

$v_1$ = 2 m/s, $\Delta t_1$ = 3 s ; $v_2$ = 4 m/s $\Delta t_2$ = 2 s

The v-t graph for the squirrel is shown on the left. Since the displacement is the area under the curve, we draw rectangles under the v-t graph as shown in the second figure.



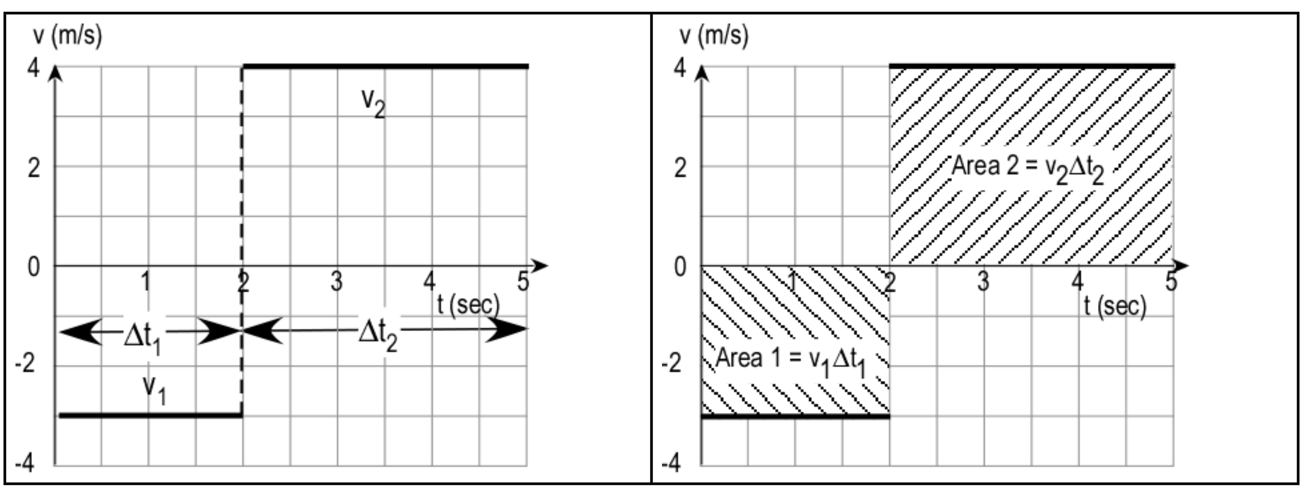

From the area under the v-t graph, the displacement is

$\Delta x$ = Area 1 + Area 2 = $v_1 \Delta t_1 + v_2 \Delta t_2$
= 2x3 + 4x2 = 6 + 8 = 14 m.
The squirrel travels a total of 14m.

**Example 2**. A cat walks backward from its starting point with a speed of 3 m/s for 2 seconds, then runs forward at 4 m/s for 3 seconds. Calculate the cat's displacement using a graphical method.

$v_1$ = –3 m/s, (negative because it walks backward) $\Delta t_1$ = 2 s ; $v_2$ = 4 m/s $\Delta t_2$ = 3 s

The v-t graph is shown on the left. The area under the v-t graph is shown in the second figure.




From the area under the v-t graph, the displacement is

$\Delta x$ = Area 1 + Area 2 = $v_1 \Delta t_1 + v_2 \Delta t_2$
= –3x2 + 4x3 = –6 + 12 = 6 m.
The cat's displacement is 6 m in the forward direction.