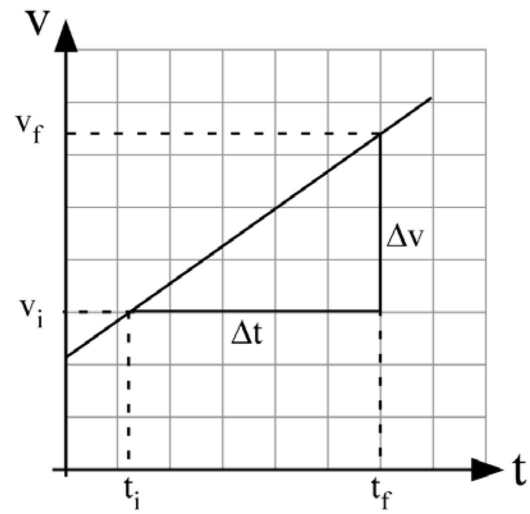
Reading Page: Acceleration

(Review from Unit II)

Acceleration occurs when the velocity of an object changes with time. Acceleration (symbol: a) is given by the formula

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time over which the change in velocity occurred}}$$

Or, from the slope of the graph, we can write the acceleration to be



$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

$$a\left(t_f - t_i\right) = v_f - v_i$$

$$a\Delta t = v_f - v_i$$

$$a\Delta t + v_i = v_f$$

Which we can rearrange as

$v_f = v_i + a\Delta t$ … Motion Equation # 1

Notice that Motion Equation #1 is similar to the equation for a straight line,

$y = mx + b$, where y = velocity v, m = acceleration a, x = change in time Δt, and b = initial velocity, v_i.

Since the v *vs* t graph is a straight line, its equation can be written in the form

$v_f = a\Delta t + v_i$ where

v_f is the velocity variable

Δt is the time variable

v_i is the intercept of the line, or the velocity at time t = 0, the initial or starting velocity

a = the slope, $\Delta v/\Delta t$, which is the acceleration

Here v_f is the final velocity, v_i is the initial velocity, Δt is the time interval over which the velocity changes from v_i to v_f and a is the acceleration. The slope of the v *vs* t graph indicates the rate of change of velocity with time. A slope is 2 m/s^2 means that for every second the object is traveling, its velocity increases by 2 m/s.

Units of acceleration:

Since Δv is in units of velocity, length/time; Δt is in units of time, therefore acceleration:

$$a = \frac{\Delta v}{\Delta t} = \frac{\text{length/time}}{\text{time}} = \frac{\text{length}}{\text{(time)(time)}} = \frac{\text{length}}{\text{time}^2}$$

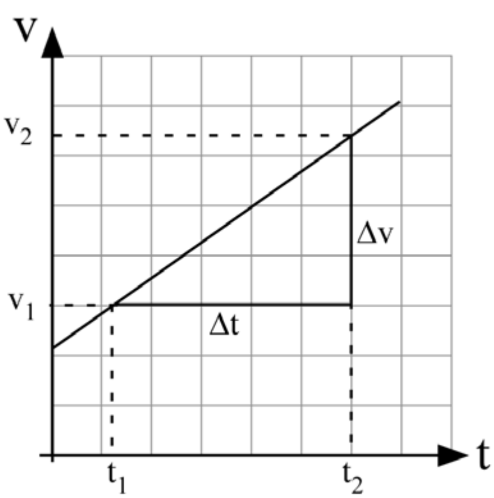
Example: A car accelerates at a steady rate from an initial velocity of 3 m/s to a final velocity of 11 m/s in a time of 4 sec. Calculate its acceleration a) mathematically, and b) graphically.

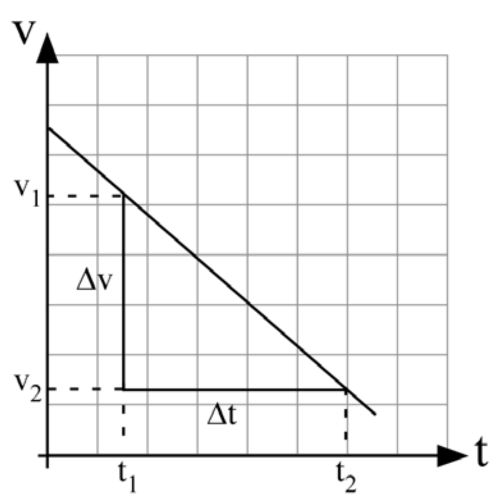
a) Since v_i = 3 m/s, v_f = 11 m/s and t = 4 s, the acceleration is

$$a = \frac{v_f - v_i}{\Delta t} = \frac{(11 - 3)\ \text{m/s}}{4\ \text{s}} = 2\ \frac{\text{m}}{\text{s}^2}$$

b) What does the value $2\frac{\text{m}}{\text{s}^2}$ mean? It might make more sense if we write it as 2 *m/s/s* (read as 2 meters per second per second). This means that for *every* second that goes by, the car's velocity increases by 2 *m/s*. In the first second of the car's motion, its velocity changes (increases) by +2 *m/s*, from a velocity of 3 *m/s* to 5 *m/s*. In the second second, it increases again from 5 *m/s* to 7 *m/s*. In the third second, it increases yet again, from 7 *m/s* to 9 *m/s*. And in the fourth and final second, it increases from 9 *m/s* to 11 *m/s*. In other words, the change in the car's velocity is 2 *m/s per second*.

In the two examples discussed above, all velocities are positive. The slope, which is the acceleration, can be positive or negative. If an object speeds up, as when a train speeds up, its final velocity v_f is more than its initial velocity v_i, the slope of the graph is positive and a is positive (left graph). If an object slows down, as when a car is braking, v_f is less than v_i, (Δv<0) the slope is negative and therefore a is negative (right graph).





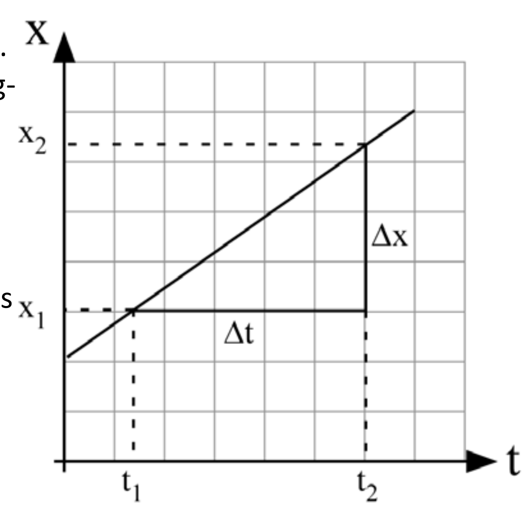
Left: in this v-t graph, the value of v increases with time, giving a positive slope.

Right: in this v-t graph, the value of v decreases with time, giving a negative slope.

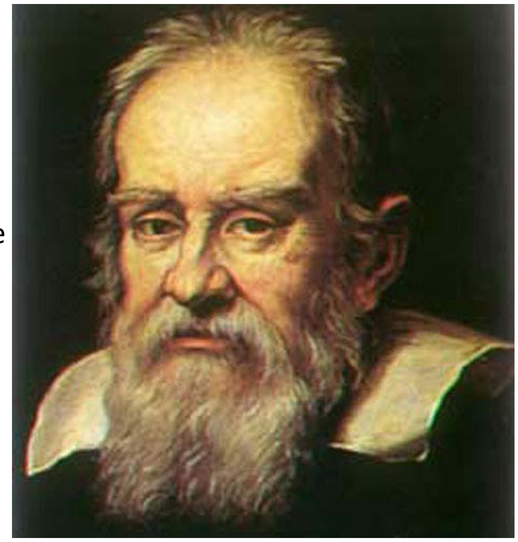
Straight-line graphs

We have seen straight-line graphs before in uniform motion. When we have straight-line *x vs t* graphs, the position changes uniformly with time, and the *rate of change of position* gave us the *average velocity.*

In this unit, we see that the *v vs t* graphs are straight lines – namely, the velocity changes uniformly with time. In an analogous fashion, the *rate of change of velocity with time* is the *average acceleration*.



Reading Page: What is Acceleration due to Gravity?



What is gravity? "It is a force." "It makes things fall down." Have you ever heard those responses? Yes, gravity is a force that makes things come down. When we are thinking about motion, though, we deal primarily with the *acceleration* due to gravity, usually given the symbol *g*.

Before the time of Galileo Galilei* (1564 –1642) most people thought that heavier objects fell faster than light objects. After all, heavier objects were more "weighty" and should fall faster. It took a long series of experiments for Galileo to prove that the mass of an object had nothing to do with the rate at which objects fell.

Galileo's claim was that if you drop an object, it gains speed and moves faster and faster as it travels toward earth. What's more, the rate at which it speeds up, the acceleration, is the *same* all the time, and for all objects.

What did Galileo mean? Imagine that you are driving in a car race on a long stretch of very straight, flat highway. You start your car, and push on the accelerator – but here's the difference – there's not a lot a friction on the road surface to slow you down, and you keep the accelerator pressed at the same level so that the acceleration is always the <u>same</u>. What happens? Your car starts moving, slowly at first, then picks up speed and goes faster (and faster and faster…). At some point, the little friction that the road has will prevent the car from speeding up any more, and the car will reach a steady speed.

When an object is dropped, a similar thing happens. It starts out moving slowly, then picks up speed. The rate at which it speeds up, namely, the acceleration, is the same at all times (similar to keeping the gas pedal pressed at a fixed place). The longer the object travels, the more speed it gains. It would gain speed forever, if it weren't for air resistance. Air resistance acts like friction. For many objects, like balls and expensive coffee cups, air resistance is extremely small and does not play a role until the object reaches high speeds. For light objects that are also large, like a sheet of paper, air resistance kicks in at pretty low speeds, so they fall at a different rate than the same piece of paper wadded up into a ball.

On Earth, the acceleration due to gravity is g = –9.8 m/s^2 (average[#]). Remember: an acceleration of –9.8 m/s^2 means that for *every second* the object travels it picks up a velocity of –9.8 m/s. That's what 9.8 m/s *per second* means! Since velocities increase at a steady rate with time, an object that is dropped from a tall tower with zero velocity has a velocity of –9.8 m/sec one second later (negative because the velocity points downward!). Two seconds later, it gains an additional –9.8 m/s for a total velocity of –19.6 m/s (see table). Every second that goes by makes the object pick up an additional –9.8 m/s in velocity. Of course, because its velocity is constantly increasing, it travels a larger distance during the fourth second than it did in the third second.

Velocity of an object dropped on earth	
Time, t (s)	Velocity v_y (m/s)
0	0
1	-9.8
2	-19.6
3	–29.4
4	–39.2

How does acceleration due to gravity affect things that are thrown upward? Since the acceleration due to gravity always attracts objects down to earth, gravity still tries to tug the object downward. Instead of acting like an accelerator, it acts like a brake. It *slows down* objects that are thrown upward – and slows them down *at the same rate* of –9.8 m/s^2. Just as objects that fell downward gained a velocity of 9.8 m/s for each second that they fell, objects that are thrown upward *lose* 9.8 m/s for each second that they travel upward. If you throw a ball upward with a velocity of 50 m/s, one second later it will have a velocity of 50 - 9.8 = 40.2 m/s … and so on (see table).

Velocity of object thrown upward on earth	
Time, t (s)	Velocity v_y (m/s)
0	50
1	40.2
2	30.4
3	20.6
4	10.8

The value of acceleration due to gravity is not the same everywhere in the universe. On other planets its value is dictated by the mass of the planet (or star or satellite) and the distance of the object from the center of the planet. For objects that do not get too far from the planet, one can use the value at the surface of the planet. The table lists the values of g on the surface of a few heavenly bodies. (http://amesnews.arc.nasa.gov/erc/HowBig/gravity.htm)

* Isn't it neat that this one scientist is always referred to by his first name?

[#]The acceleration due to gravity on Earth is slightly different at the poles than at the equator due to the slight flattening of the planet at the poles. It is also different at different parts of the earth due to the presence of rocks of different densities. These variations are of the order of less than three parts in a thousand.

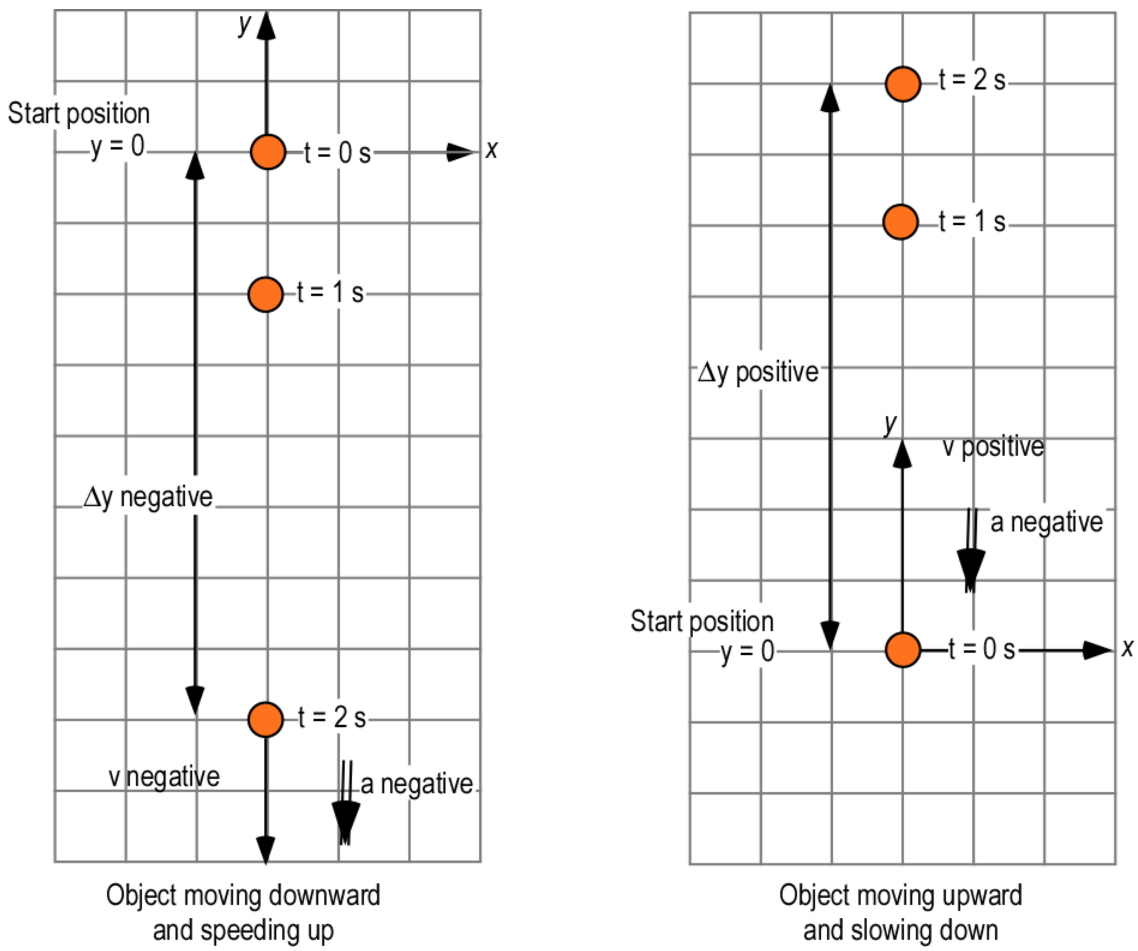
Name of planet or satellite	"g" at surface (m/sec^2)
Mercury	3.8
Venus	8.9
Earth	9.8
Mars	3.7
Jupiter	28.4
Saturn	10.8
Uranus	8.6
Neptune	11.2
Pluto	0.05
Moon	1.62

Reading Page – Motion under Gravity

Note: You may wish to use this Reading Page for teacher information, and discuss the problems in your class as appropriate

Sign convention

Motion under gravity is described by the same equations of motion discussed before. The only difference is that now the acceleration is due to gravity. After all, the force of gravity is the only force acting on the object. The acceleration a is equal to g = -9.8 m/s^2 (or -980 cm/s^2), and it ***always*** points in the downward direction. We have to be careful to define "up" and "down." It is simplest if we set the starting point as y = 0.



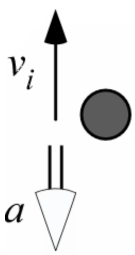
If the object is moving in the downward direction, the displacement Δy is downward, so it is negative. The velocity v points down, so it is negative; if the acceleration points down (and gravity does) it is negative too. Since the velocity and acceleration point in the same direction, the ball speeds up.

If the object is moving upward, the displacement is upward, so Δy is positive (again, the starting point is y = 0). The velocity points up, so it is positive too. Since the velocity and acceleration point in the opposite directions, the ball slows down.

Equations of motion

The equations of motion are truly one-dimensional. They can deal with up-down or left-right motion. All the factors in the equations, Δy, v_i, v_f, and a refer to one dimension. If we are measuring displacement in the up-down direction, v_i, v_f and a should also be along the up-down direction. The same holds for left-right or north-south or any other one-dimensional axis.

In all problems that involve vertical motion, the acceleration is implicit: we have to use the acceleration due to gravity, g = -9.8 m/sec^2 (-980 cm/s^2), pointing in the downward direction.



Equations of Motion:

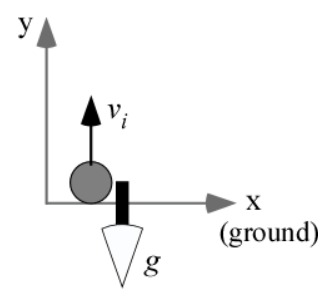
Equation # 1: $v_f = v_i + a\,\Delta t$

Equation # 2: $\Delta y = v_i \Delta t + \frac{1}{2} a(\Delta t)^2$

Equation # 3: $v_f^2 - v_i^2 = 2a\Delta y$

	Unit system	
	cm-g-s:	*SI*
Δy = distance	cm	m
Δt = time	sec	sec
v_i = initial speed	cm/sec	m/sec
v_f = final speed	cm/sec	m/sec
a= acceleration	cm/sec^2	m/sec^2

Example 1: A rock is thrown vertically up with a velocity of 25 m/sec. Calculate its velocity after 1 second.



Units: meter-kg-sec
Givens and Unknowns:
Initial velocity v_i = 25 m/s   Time Δt = 1 s   Acceleration a= –9.8 m/s^2
Final velocity v_f = ?

Notice that we write the acceleration as –9.8m/sec^2. The *negative* sign denotes that it points *downward*. In contrast, the velocity, which is in the upward direction is written as positive, +25 m/s.

When we do problems where the direction is involved, as in this problem, all factors that point upward – upward distances, upward velocities and upward accelerations have a positive sign. Similarly, all downward factors – downward distances, downward velocities, and downward accelerations, have a negative sign. Sticking with this convention keeps our directions defined and takes away any uncertainty.

Identify formulae and calculate:

Since the factors needed are v_i, v_f, a and Δt, the first equation of motion should serve us well:

$$
\begin{aligned}
v_f &= v_i + a\Delta t \\
&= 25 + (-9.8)(1) \\
&= 15.2\,\mathrm{m/s}
\end{aligned}
$$

Unit check:

$v_f = v_i + a\Delta t$

$$\left[\frac{m}{s}\right] = \left[\frac{m}{s}\right] + \left[\frac{m}{s^{\not 2}} \cdot \not s\right]$$

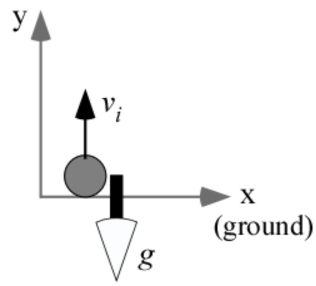
$$\left[\frac{m}{s}\right] = \left[\frac{m}{s}\right] + \left[\frac{m}{s}\right]$$

All terms on the right side have units of $\left[\frac{m}{s}\right]$.

The velocity after 1 sec is v_f = 15.2 m/sec. Notice that the velocity is less than when the rock started out. Since the velocity and the acceleration are in opposite directions, the rock slows down. Notice also that the velocity is positive. The rock, therefore, is still traveling upward.

Example 2. The rock is again thrown vertically up with a velocity of 25 m/sec. Calculate its velocity after 2 seconds.

Units: meter-kg-sec
Givens and Unknowns:
Initial velocity v_i = 25 m/s
Time Δt = 2 sec
Acceleration a= -9.8 m/s²
Final velocity v_f = ?



Unit check:

$v_f = v_i + a\Delta t$

$\left[\frac{m}{s}\right]=\left[\frac{m}{s}\right]+\left[\frac{m}{s^{2}}\bullet s\right]$

$\left[\frac{m}{s}\right]=\left[\frac{m}{s}\right]+\left[\frac{m}{s}\right]$

All terms on the right side have units of $\left[\frac{m}{s}\right]$.

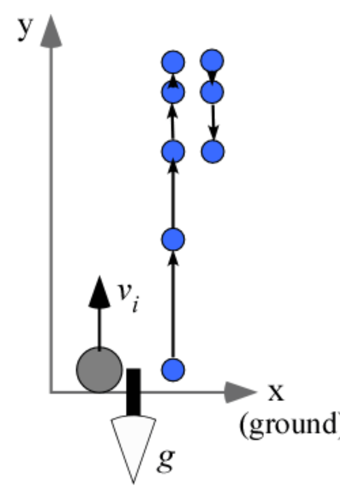
Identify formulae and calculate:

$$\begin{aligned} v_f &= v_i + at \\ &= 25+(-9.8)(2) \\ &= 5.4\,m/s \end{aligned}$$

The velocity after 2 sec is v_f = 5.4 m/s. Notice that the velocity is even less than at 1 sec (in example 1): the rock has slowed down further. The velocity is still positive, indicating that the rock is still traveling upward.

Example 3. The rock is again thrown vertically up with a velocity of 25 m/sec. (a) Calculate its velocity after 3 seconds. (b) Draw a motion diagram.

Units: meter-kg-sec
Givens and Unknowns:
Initial velocity v_i = 25 m/sec
Time Δt = 3 s
Acceleration a= -9.8 m/s2
Final velocity v_f = ?



Identity formulae and calculate:

$$\begin{aligned} v_f &= v_i + a\Delta t \\ &= 25+(-9.8)(3) \\ &= -4.4\,m/s \end{aligned}$$

The velocity after 3 sec is v_f = -4.4 m/sec. Notice that the velocity is now negative: the rock is traveling downward! The rock went up, slowing down all the time, reached the highest point and is now on its way down. Notice that nothing changed in our method of calculation. All we did was to keep the directions of the starting values (here initial velocity and acceleration) straight, and the direction of the final velocity took care of itself! That's the power of using the + and – signs to indicate directions.

Unit check:

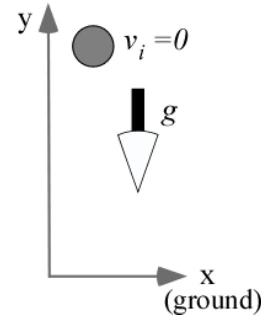
$v_f = v_i + a\Delta t$

$\left[\frac{m}{s}\right]=\left[\frac{m}{s}\right]+\left[\frac{m}{s^{2}}\bullet s\right]$

$\left[\frac{m}{s}\right]=\left[\frac{m}{s}\right]+\left[\frac{m}{s}\right]$

All terms on the right side have units of $\left[\frac{m}{s}\right]$.

Example 4. A bird's egg falls from a tree that is 2000 cm high. How much time does it take to hit the ground?



Units: cm-g-sec
Givens and Unknowns:
Initial velocity v_i = 0 ("drops" or "falls from" implies that the initial velocity is zero)
Δy = -2000 cm (We take the point of drop as y = 0; the egg drops *downward*, so its displacement Δy is *negative*)
Acceleration a= -980 cm/s^2 Time Δt = ?
Identify formulae and calculate:

$$\Delta y = v_i \Delta t + \frac{1}{2} a (\Delta t)^2$$
$$-2000 = 0 + \frac{1}{2}(-980)(\Delta t)^2$$
$$-2000 = -490(\Delta t)^2$$
$$\frac{2000}{490} = (\Delta t)^2$$
$$(\Delta t)^2 = 4.08$$
$$\Delta t = \sqrt{4.08}$$
$$\Delta t = 2.02 \text{ sec}$$

Unit check:

$$\Delta y = v_i \Delta t + \frac{1}{2} a (\Delta t)^2$$
$$[\text{cm}] = \left[\frac{\text{cm}}{\cancel{\text{s}}} \bullet \cancel{\text{s}}\right] + \left[\frac{\text{cm}}{\cancel{\text{s}^2}} \bullet \cancel{\text{s}^2}\right]$$
$$[\text{cm}] = [\text{cm}]$$

Consistent units are $\frac{\text{cm}}{\text{s}}$ for velocity, $\frac{\text{cm}}{\text{s}^2}$ for acceleration and s for time.

The egg takes 2.02 sec to fall and hit the ground. Notice that the distance Δy has a negative value, -2000 cm. However, as the calculation proceeded, the negative signs cancelled out, giving us a positive number inside the square-root sign. If you find yourself trying to take the square root of a negative number, then this is a sure indication of an error.

Example 5. Calculate the velocity of the egg in Example 4 when it hit the ground.

Units: cm-g-sec
Givens and Unknowns:

Initial velocity v_i = 0
Distance Δy = -2000 cm
Acceleration a= -980 cm/s^2
Final velocity v_f= ?
Equation 3 connects v_i, v_f, a and Δy.
Identify formulae and calculate:

$$v_f^2 - v_i^2 = 2a\Delta y$$
$$v_f^2 = v_i^2 + 2a\Delta y$$
$$v_f^2 = 0^2 + 2(-980) \times (-2000)$$
$$v_f^2 = +3920000$$
$$v_f = \sqrt{3920000} = 1980 \text{cm/ s}$$

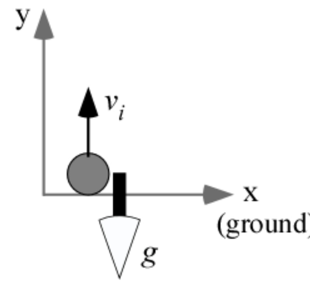
Unit check:

$$v_f^2 = v_i^2 + 2a\Delta y$$
$$v_f = \sqrt{v_i^2 + 2a\Delta y}$$
$$\left[\frac{\text{cm}}{\text{s}}\right] = \sqrt{\left[\frac{\text{cm}}{\text{s}}\right]^2 + \left[\frac{\text{cm}}{\text{s}^2} \bullet \text{cm}\right]}$$
$$\left[\frac{\text{cm}}{\text{s}}\right] = \sqrt{\left[\frac{\text{cm}}{\text{s}}\right]^2 + \left[\frac{\text{cm}^2}{\text{s}^2}\right]} = \sqrt{\left[\frac{\text{cm}}{\text{s}}\right]^2}$$
$$\left[\frac{\text{cm}}{\text{s}}\right] = \left[\frac{\text{cm}}{\text{s}}\right]$$

Consistent units are $\frac{\text{cm}}{\text{s}}$ for velocity, $\frac{\text{cm}}{\text{s}^2}$ for acceleration and s for time.

The final velocity of the egg as it hits the ground is 1980 cm/s. Splat!

Example 6. A tennis ball is vertically thrown up with a velocity of 4.2 m/s. How much time does it take to reach the highest point?



Units: meter-kg-sec
Givens and Unknowns:
Initial velocity v_i = +4.2 m/s (positive implies that the initial velocity is upward)
Acceleration a= -9.8 m/s²
Final velocity v_f=0 (at the top the object has zero velocity – which is why it cannot go any higher!)
Time Δt = ?

Identify formulae and calculate:
The formula we need must connect v_i, v_f, a and Δt: Equation #1.

$$v_f = v_i + a\Delta t$$
$$a\Delta t = v_f - v_i$$
$$\Delta t = \frac{v_f - v_i}{a}$$
$$\Delta t = \frac{0 - 4.2}{-9.8}$$
$$\Delta t = 0.43\text{sec}$$

$$v_f = v_i + a\Delta t$$
$$\Delta t = \frac{v_f - v_i}{a}$$
$$[s] = \left[\frac{\frac{m}{s}}{\frac{m}{s^2}}\right] = \left[\frac{m\, s^2}{s\, m}\right]$$
$$[s] = [s]$$
All terms on the right side have units of s.

The ball takes 0.43 s to reach the highest point.

Example 7. A tennis ball in Example 6 again thrown vertically up with a velocity of 4.2 m/s. How high is it at the highest point?

Units: meter-kg-sec

Givens and Unknowns:
Initial velocity v_i = +4.2 m/s (positive implies that the initial velocity is upward)
Acceleration a= -9.8 m/sec²
Final velocity v_f=0 (at its highest point the object has zero velocity – which is why it cannot go any higher!)
Distance Δy = ?

Identify formulae and calculate:
The formula must connect v_i, v_f, a and Δy: Equation #3.

$$v_f^2 - v_i^2 = 2a\Delta y$$
$$\Delta y = \frac{v_f^2 - v_i^2}{2a}$$
$$\Delta y = \frac{0^2 - (4.2)^2}{2(-9.8)}$$
$$\Delta y = \frac{-17.64}{-19.6} = +0.90\text{m}$$

$$v_f^2 = v_i^2 + 2a\Delta y$$
$$v_f = \sqrt{v_i^2 + 2a\Delta y}$$
$$\left[\frac{m}{s}\right] = \sqrt{\left[\frac{m}{s}\right]^2 + \left[\frac{m}{s^2}\bullet m\right]}$$
$$\left[\frac{m}{s}\right] = \sqrt{\left[\frac{m}{s}\right]^2 + \left[\frac{m^2}{s^2}\right]} = \sqrt{\left[\frac{m}{s}\right]^2}$$
$$\left[\frac{m}{s}\right] = \left[\frac{m}{s}\right]$$
Consistent units are $\frac{m}{s}$ for velocity, $\frac{m}{s^2}$ for acceleration and s for time.

The ball reaches a height of 0.9m.

Reading Page: Motion in Two Dimensions - I

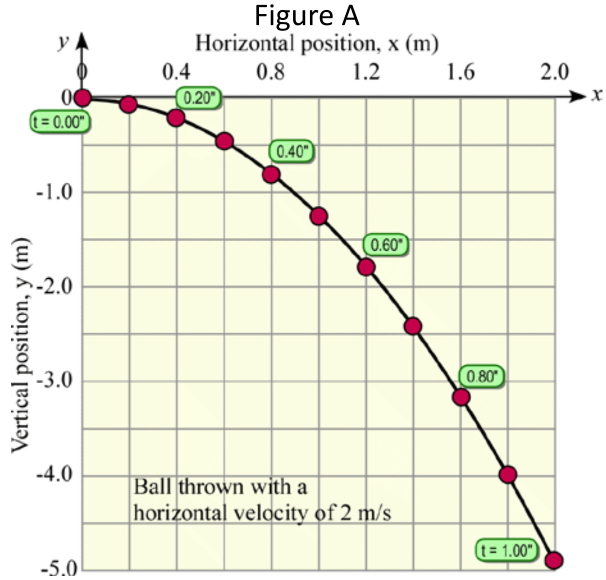
When you throw a ball with a sideways velocity, it travels sideways, but it goes down as well. That's what we call two-dimensional motion. When we analyze motion in two dimensions, we need to keep track of both the horizontal and the vertical motion.

Figure A shows the trajectory of a ball thrown with a horizontal velocity of 2 m/s. The ball's position is plotted every 0.1 sec (see time flags). The path is similar in shape to the trajectory you just traced in the *Trace the Trajectory Lab*. Notice that the vertical displacement is plotted on the negative axis, to indicate that the ball is traveling downward.

It is useful to track the horizontal motion and the vertical motion separately using motion diagrams (figure B). Let's first look at the horizontal motion. Since the trajectory is already plotted at equal time intervals (0.1 s apart), we could just look at the horizontal part of the motion by tracing the rightward motion along the horizontal axis.

The horizontal motion looks just like a motion diagram for uniform motion! The position dots are equally spaced, and the velocity arrows are all the same length. This should not be surprising. There is no acceleration (or deceleration) along the horizontal direction, so the velocity should not change – and that means uniform motion.

Figure A



Let's look at the motion along the vertical direction, figure C. We'll use the same idea: since the ball's position is already marked at equal time intervals, we can trace the downward motion along the vertical axis to give us the motion diagram. The dots get farther apart, so the ball is speeding up. The velocity arrows get longer and longer, indicating that the velocity in the downward direction is continuously increasing – after all, the ball is falling, so it should speed up.

Figure B

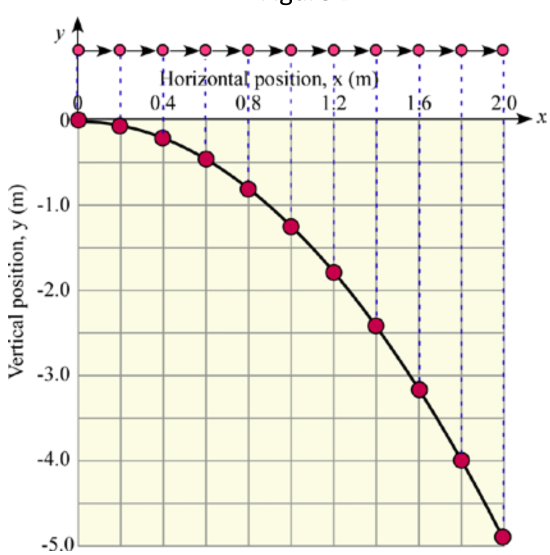
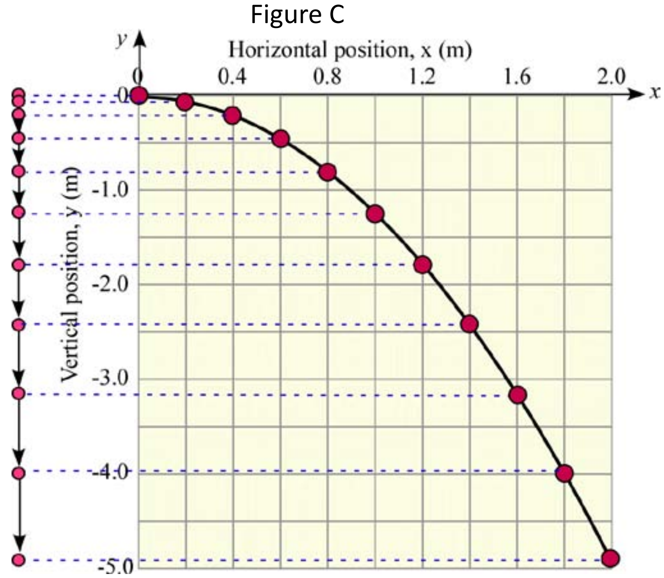


Figure C



Comparing two balls

Now let's compare the trajectories of a ball thrown with a horizontal velocity with one that is dropped straight downward.

Figure D. shows the trajectories of two balls – one thrown with a horizontal velocity of 2 m/s and one dropped straight down from rest. Next, we draw their motion diagrams – both for horizontal motion and for vertical motion, shown in Fig. E.

Figure D

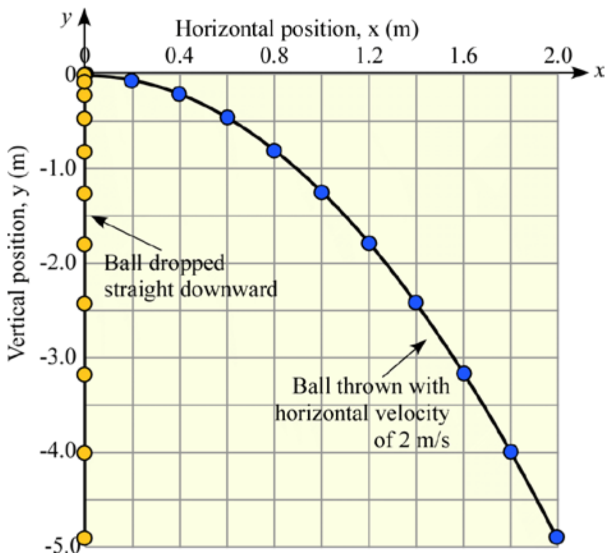
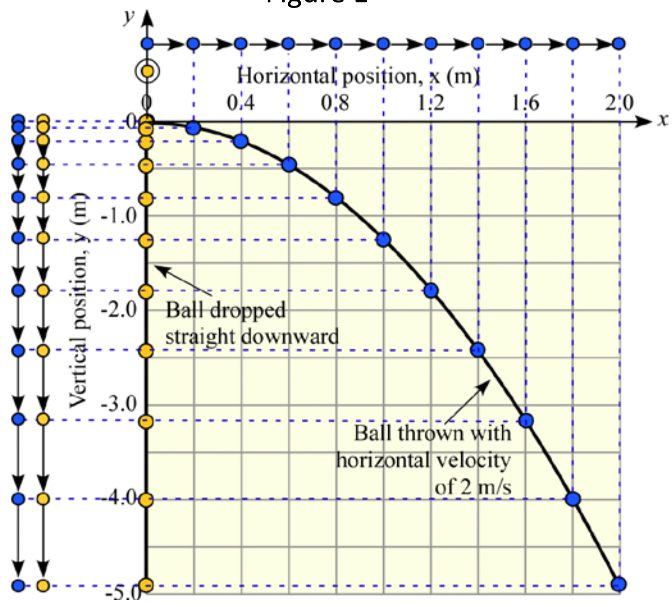


Figure E



Notice that the horizontal motion for the "dropped straight down ball" is just a spot that does not change position – since the ball does not travel in the horizontal direction at all.

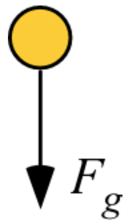
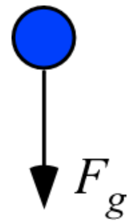
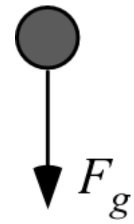
The motion diagram along the vertical direction is interesting – the vertical motion diagrams for both balls are *exactly* alike. At every instant in time, both balls are at exactly the same vertical height below the starting point. Moreover, their vertical velocities at any specific instant in time (e.g., 0.6 s) are *exactly the same*.

This should really not surprise us. After all, both balls fall at the same rate (both experience the same acceleration, g). Since both started with a vertical velocity that is zero, and they speed up at the same rate (g), they should have the same vertical velocity at any given instant in time – and that means identical vertical motion diagrams.

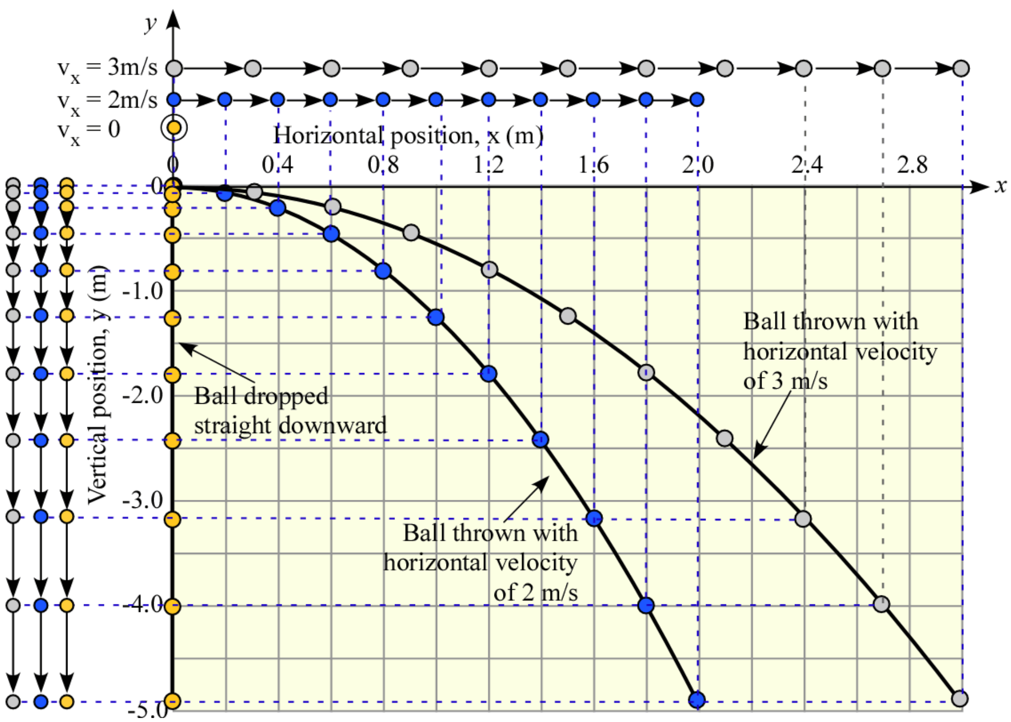
Comparing Three Balls

What would one expect if we compared three balls: one dropped down from rest, a second one thrown with a horizontal velocity of 2 m/s, and the third thrown with a horizontal velocity of 3 m/s? In figure F are the trajectories and the motion diagrams for all three balls. The trajectory for the fast 3 m/s ball goes wider – it travels a larger horizontal distance, but its motion in the downward direction is exactly the same as that of the ball dropped straight down or the ball thrown with a horizontal velocity of 2 m/s. At any given instant in time, all three balls are at the same height, since they started from the same spot, and had the same amount of vertical velocity when they started, namely, zero.

Why is this so? Because the forces felt by the three balls is due to gravity. Check out their force diagrams, and the accelerations that arise from them:

F. *Force Diagrams for three balls:*		
Ball of mass M dropped straight down:	Ball of mass M thrown with horizontal velocity of 2 m/s:	Ball of mass M thrown with horizontal velocity of 3 m/s:
 *Force* $F = Mg$ *Vertical acceleration* $a_y = \frac{Force}{Mass} = \frac{F}{M} = \frac{Mg}{M} = g$	 *Force* $F = Mg$ *Vertical acceleration* $a_y = \frac{Force}{Mass} = \frac{F}{M} = \frac{Mg}{M} = g$	 *Force* $F = Mg$ *Vertical acceleration* $a_y = \frac{Force}{Mass} = \frac{F}{M} = \frac{Mg}{M} = g$

Motion of three balls:



The key difference between the vertical and the horizontal parts of the motion shown in Figure G. is that the vertical motion is affected by gravity, while the sideways motion is *not*. In other words, the velocity in the vertical direction is speeded up (on the way down) by the acceleration due to gravity. [If the object were thrown so that it went upward, it would be slowed down by gravity on its way up – and we will examine such examples later]. The horizontal motion just continues on, unaffected by accelerations for the most part (unless we are traveling so fast that air resistance kicks in to cause a deceleration).

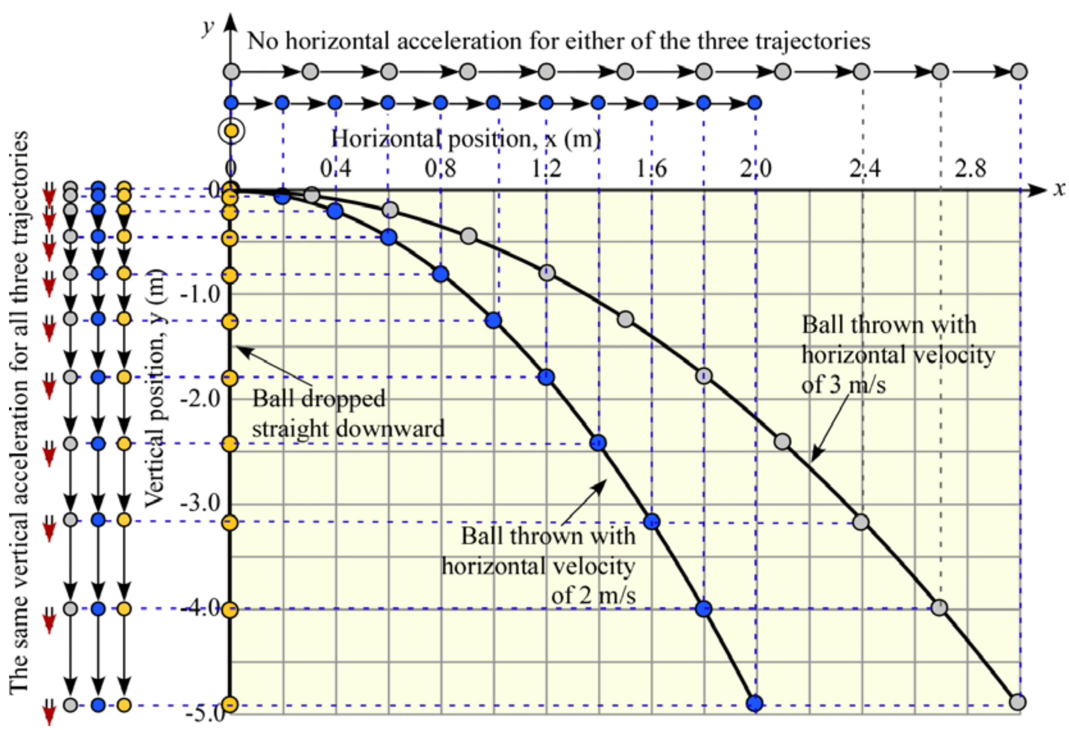
The fact that the vertical motion is affected by gravity has another profound effect. All vertical motion, regardless of the horizontal part is affected in the same way. Notice the vertical motion of the three balls in the example above. All three balls had a zero initial vertical velocity, although they had varying amounts of horizontal velocity. All fell at the same rate – meaning that at *any* particular clock reading, the three of them had *traveled the same vertical distance*. No wonder their vertical motion diagrams look identical!

The fact that all vertical motion is affected by gravity has another profound consequence: vertical motion determines how much time it takes for the object to hit the ground. [Since the ground is vertically below the object, it must be the vertical velocity that determines this – it could not possibly be the horizontal velocity since the horizontal velocity just makes the object travel along the horizontal direction]. During the time period Δt that the object is traveling downward, the horizontal motion carries the object sideways.

So the vertical direction decides the time taken to get to the ground, and the horizontal velocity decides how far across it gets during that time.

What about the acceleration?

The motion diagrams above tell us about the acceleration along the horizontal and along the vertical. Along the horizontal, all three trajectories have uniform velocities – so the acceleration is zero (shown in Figure H). Along the vertical, all three trajectories have the same vertical velocities at every instant. Since the acceleration due to gravity, g is the same for all three, their velocities all increase at the same rate. [since acceleration and velocity point in the same direction, the objects speed up.]



Parameters and equations that apply to two-dimensional trajectories:	
v_{ix} is the initial horizontal velocity v_{iy} is the initial vertical velocity v_{fx} is the final horizontal velocity v_{fy} is the final vertical velocity Δt is the time a_x is the horizontal acceleration (usually = 0) a_y is the vertical acceleration (usually = g)	
Horizontal motion: $v_{fx} = v_{ix}$ (since $a_x = 0$) $\Delta x = v_{ix}\Delta t$	Vertical motion: $v_{fy} = v_{iy} + a_y\Delta t$ $\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$ Since $a_y = g = -9.8 m/s^2$ in SI units, $v_{fy} = v_{iy} + (-9.8)\Delta t$ $\Delta y = v_{iy}\Delta t + \frac{1}{2}(-9.8)(\Delta t)^2$

An alternate diagram that illustrates the horizontal and vertical velocities is shown in Figure I.

The ball starts with an initial horizontal velocity, which does not change since there is no acceleration in the horizontal direction. In the figure this is indicated by the horizontal arrow for the velocity vector always being the same length. Along the vertical direction, the ball has zero velocity initially. However, gravity makes the ball pick up velocity in the vertical direction – so we start with a vertical velocity vector that is zero, but increases as time passes. Meanwhile the ball continues to move horizontally. The effect is that the ball has a constant horizontal velocity and an accelerating vertical velocity.

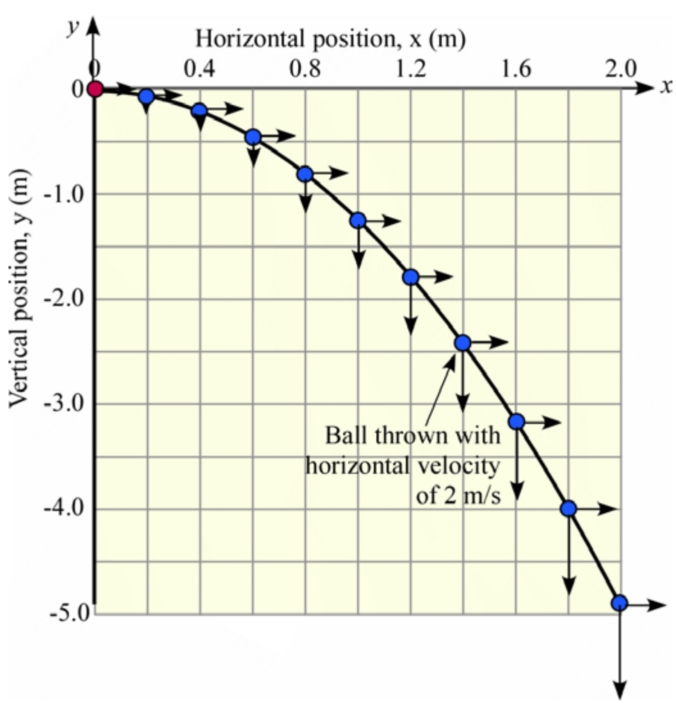


Figure I

Slope of a trajectory curve:

We have frequently calculated the slopes of motion curves – usually curves that have t on the horizontal axis. Note that these trajectory curves have y and x on the axes. The slope does *not* give us a velocity! Instead, it gives us the direction of motion – which is the direction of the *total* velocity. It also gives us the ratio of the vertical velocity to the horizontal velocity.

Powerpoint: Horizontal Trajectory ppt

Why is it useful or necessary to have vectors?

Reading Page: 2D Motion Calculations

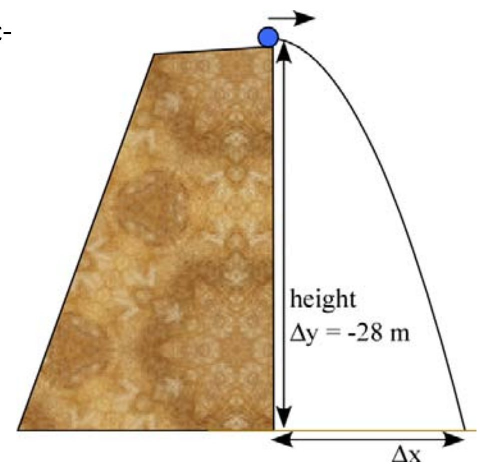
Since projectile motion can be neatly divided into motion along the horizontal and vertical directions, we can treat them as two separate one-dimensional motions.

The *vertical* motion involves the acceleration due to gravity, g, the initial upward or downward velocity, v_{iy}, the amount of time the ball travels before it hits the ground, Δt, and the vertical distance traveled, Δy. The *horizontal* motion involves the initial horizontal velocity, v_{ix} zero acceleration, the amount of time the ball travels before it hits the ground, Δt, and the horizontal distance traveled, Δx. The *only common factor* between the vertical and horizontal motion is the time of travel Δt!

We see below how we apply equations for one-dimensional motion for the vertical motion and the horizontal motion separately.

Example 1. A ball is thrown horizontally from a cliff with a velocity of 5 m/sec. If the cliff is 28 m tall,How much time does the ball take to hit the ground below?

d) How far from the cliff does it land?



Solution:

Units: SI units

Draw diagram and mark salient data:

Givens and Unknowns:

(a) Vertical motion:

Initial vertical velocity v_{iy} = 0
Vertical acceleration a_y = -9.8 m/s^2
Vertical distance Δy = -28 m
Time Δt = ?

Identify formulae and calculate:

Equation of motion #2 should serve us well:

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

$$-28 = 0 + \frac{1}{2}(-9.8)(\Delta t)^2$$

$$-28 = -4.9(\Delta t)^2$$

$$(\Delta t)^2 = \frac{28}{4.9} = 5.71$$

$$\Delta t = \sqrt{5.71} = 2.4\,\text{sec}$$

The ball takes 2.4 sec to hit the ground.

Unit check:

$$\Delta x = v_i\Delta t + \frac{1}{2}a(\Delta t)^2$$

$$[\text{cm}] = \left[\frac{\text{cm}}{\cancel{\text{s}}}\bullet\cancel{\text{s}}\right] + \left[\frac{\text{cm}}{\cancel{\text{s}^2}}\bullet\cancel{\text{s}^2}\right]$$

$$[\text{cm}] = [\text{cm}]$$

Consistent units are $\frac{\text{cm}}{\text{s}}$ for velocity, $\frac{\text{cm}}{\text{s}^2}$ for acceleration and s for time.

(b) Now for the horizontal motion: While the ball was accelerating downward, it wasn't just traveling downward, it was also traveling *sideways* because of that original horizontal velocity. The time for which it traveled sideways is the same as the time it traveled down (after all, it is the same ball). Therefore:

Initial horizontal velocity v_{ix} = 5 m/s
Horizontal acceleration, a_x = 0
Time of travel Δt = 2.4 sec
Distance of horizontal travel Δx = ?

Equation #2 gives us:

$$\Delta x = v_{ix}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$
$$\Delta x = (5)(2.4) + 0$$
$$= 12\,\mathrm{m}$$

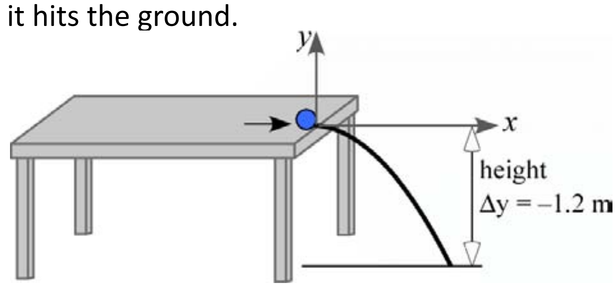
The ball travels 12 m in the horizontal direction.

Example 2: A ball rolls on a horizontal table with a velocity of 2.8 m/s. It falls off the edge. The table is 1.2 m tall.

a) How much time does it take to hit the ground?

b) Calculate its vertical velocity just before it hits the ground.

c) Draw graphs of the ball's horizontal and vertical velocities as a function of time, starting from the time it leaves the table and ending when it hits the ground.



Solution:
Units: SI units
Draw diagram and mark salient data:

Givens and Unknowns:

(a) Vertical motion:
Initial vertical velocity v_{iy} = 0
Vertical acceleration a_y = -9.8 m/s^2
Vertical distance Δy = -1.2 m
Time Δt = ?

Identify formulae and calculate:

Equation of motion #2 should serve us well :

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$
$$-1.2 = 0 + \frac{1}{2}(-9.8)(\Delta t)^2$$
$$-1.2 = -4.9(\Delta t)^2$$
$$(\Delta t)^2 = \frac{1.2}{4.9} = 0.245$$
$$\Delta t = \sqrt{0.245} = 0.495\,\mathrm{sec}$$

The ball takes 0.495 s to hit the ground.

(b). In order to calculate its vertical velocity when it hits the ground, we list the velocity information:

Initial vertical velocity v_{iy} = 0

Acceleration, a_y = -9.8 m/s^2
Final velocity v_{fy} = ?

Identify formulae and calculate:

Equation of motion #1 will help us here:

$$v_f = v_i + a\Delta t$$

For vertical motion, we can include the "y" subscripts,

$$v_{fy} = v_{iy} + a_y\Delta t$$

Substituting the values above,

$$v_{fy} = v_{iy} + (-9.8)(0.495)$$

$$= 4.85 \text{ m/s}$$

Unit check:

$v_f = v_i + a\Delta t$

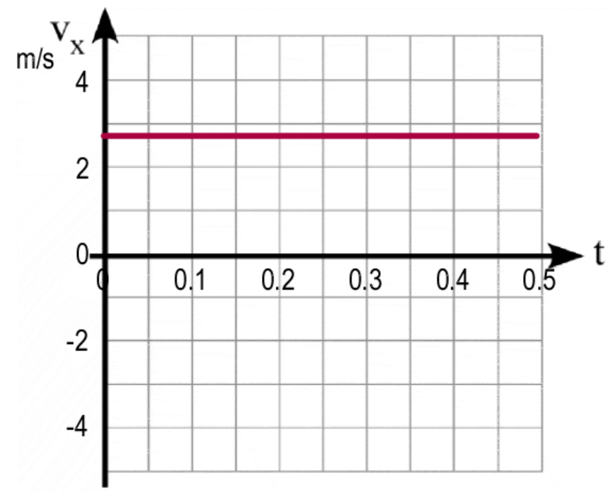
$$\left[\frac{m}{s}\right] = \left[\frac{m}{s}\right] + \left[\frac{m}{s^{\not{2}}} \cdot \not{s}\right]$$

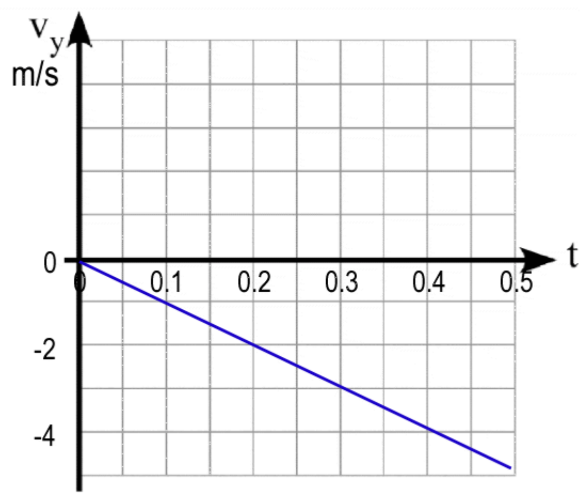
$$\left[\frac{m}{s}\right] = \left[\frac{m}{s}\right] + \left[\frac{m}{s}\right]$$

All terms on the right side have units of $\left[\frac{m}{s}\right]$.

The vertical velocity of the ball is 4.85 m/s. Note that this is only its vertical velocity – not the total velocity, since it also has a horizontal velocity.

(c) Graphs:





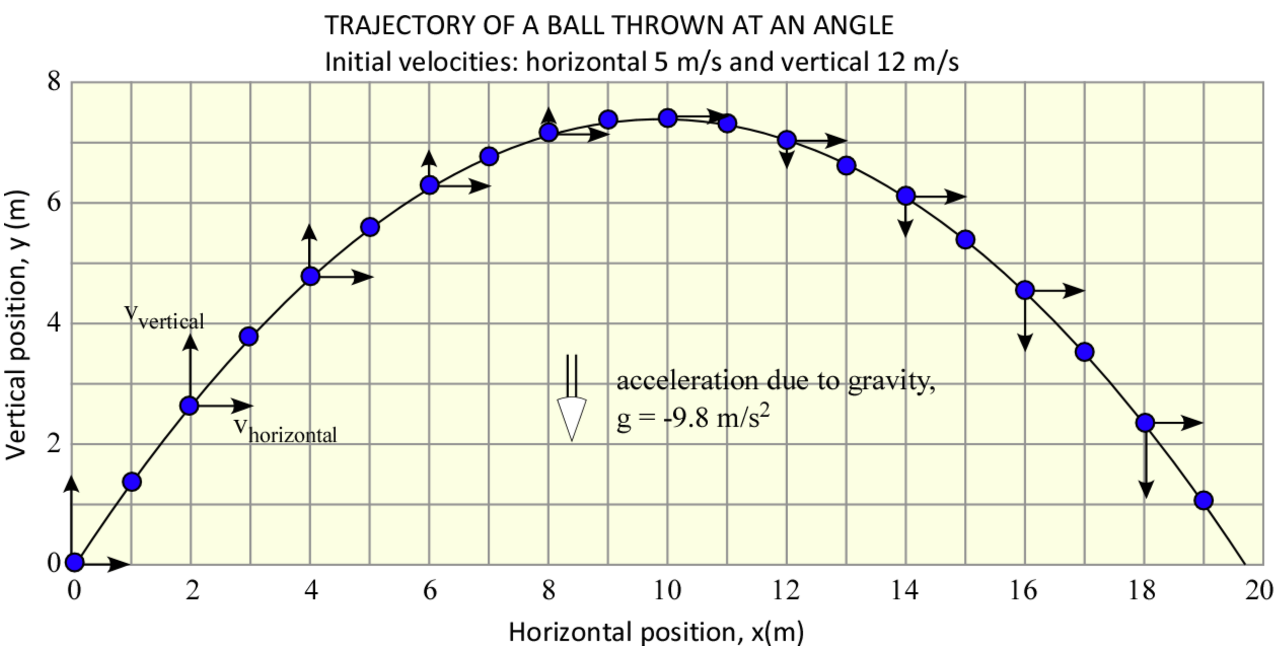
Reading Page: Motion in Two Dimensions, II

If a ball has both vertical and horizontal initial velocities, such as when a basketball is thrown into a basket, we have an initial vertical velocity that is not equal to zero. The trajectory takes on the familiar parabolic arc seen on ball fields daily.

The diagram below shows the trajectory of such an object. Just as for objects thrown with only an initial horizontal velocity, the horizontal velocity in this trajectory remains the same all through (no acceleration along the horizontal, $a_x = 0$).

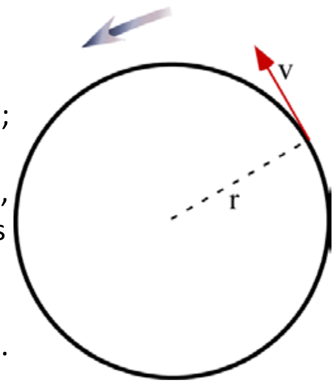
The vertical velocity behaves the same as when we throw a ball straight up. It starts with a large vertical component, and decreases steadily, slowed down by gravity. At the highest point in the trajectory it has zero vertical velocity (it continues to have a horizontal velocity, though, since that part does not change). The ball then starts to fall downward, speeded up by gravity, just as all falling objects do.

The vertical velocity has the same magnitude (or amount) at any given height, say 6 m, whether the ball is on the way up or on the way down. Why? Because the rate at which an object slows down due to gravity on the way up (a_y = -9.8 m/s^2) is the same as the rate at which it speeds up on the way down (a_y = -9.8 m/s^2). The negative sign for a_y indicates that the acceleration due to gravity points downward. This is true of balls thrown vertically upward too. Similar to the vertically thrown ball, g points downward everywhere in the path of the object.



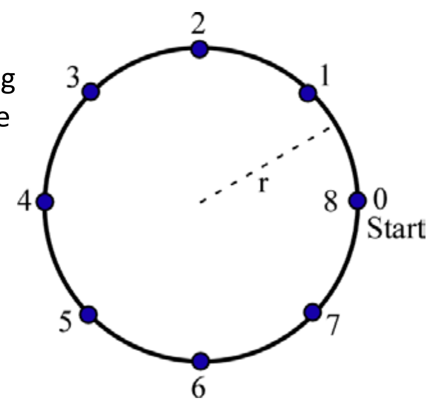
Reading Page: Describing Uniform Circular Motion

What is Uniform Circular Motion?



An object that moves with a *constant speed in a circular path* is said to have uniform circular motion. Uniform, because its speed does not change; circular because it travels in a circle of fixed radius (r). The object might be a ball tied to a string that is swung in a circle, a spaceship or planet in orbit, or a grain of sand stuck in the treads of a bicycle wheel. Now, some objects might change their speed with time, or they may have elliptical orbits - these objects are *not* in *uniform* circular motion. We restrict our study to objects that travel at a constant speed and in a circular path of fixed radius.

The diagram shows the path of an object in uniform circular motion, traveling with a constant speed v. At any point along its path, its velocity is a tangent to its path. The vector $\vec{v}$ is called the tangential velocity[1]. When the object has completed one full circle, it starts all over again, and continues to trace the circular path over and over again. Since its speed is constant (v), it travels one full circle in a specific time, T. The next time it goes around, it takes the same amount of time, T, since it travels the same distance (the circumference) at a constant speed v. The time to travel one circumference is called the period T. This is similar to the second hand of a clock, which takes exactly 60 sec to travel one full circle around the clock.



The motion diagram of this object is shown on the right. The starting position is marked "Start." For clarity, the successive positions of the object are numbered. Since the object travels with constant speed, the position dots are at equal intervals around the circle. When the object completes one full circle, at time click 8, it is back where it started. Thus motion diagrams for uniform circular motion have position dots that are evenly spaced around the circumference of a circle. In this example, the period = 8 "time clicks."

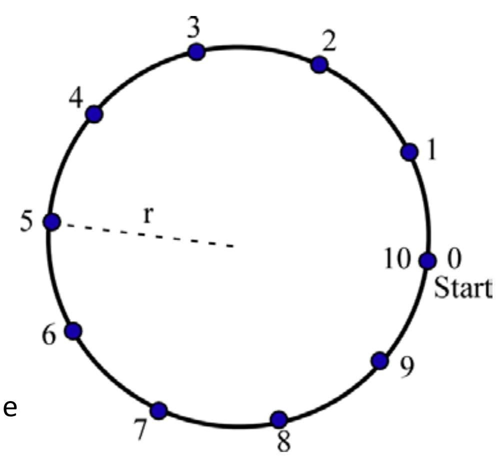
To calculate the speed we use the formula that we used for uniform motion, namely,

This expression relates speed v, radius r, and period T.

$$speed = \frac{\text{distance traveled}}{\text{time taken}}$$

$$v = \frac{circumference}{period}$$

$$v = \frac{2\pi r}{T}$$



What if the speed stays the same, but the radius increases? The distance traveled (the circumference) will be larger, so it will take longer – and the period T will increase (see figure on right). Since the speed is the same, the distance between time clicks is the same as in the previous diagram. However, since the radius is larger, it takes more time clicks (in this case, 10 clicks) to go around the circle.

---

1. As always we use "velocity" when we imply the magnitude and direction, "speed" when we refer only to the amount or magnitude.

Example 1:

A girl decides to run laps along the edge of her round trampoline. The radius of the trampoline is 2.5 m. The girl takes 18 sec to run three laps. (a) calculate the period (b) calculate her speed.

Solution:

r = 2.5 m; total time = 18 s; number of laps = 3.
(a) T=?

If she takes 18 s for 3 laps, the time taken for one lap is the period. Therefore,

$$T = \frac{\text{total time taken}}{\text{number of laps}} = \frac{18}{3} = 6\,\text{sec}$$

The period of her motion is 6 s.

(b) v = ?

$$\begin{aligned} v &= \frac{2\pi r}{T} \\ &= \frac{(2)(3.1416)(2.5m)}{6\,\text{sec}} \\ &= 2.62 m/s \end{aligned}$$

The speed of her motion is 2.62 m/s.

Frequency:

Sometimes it is convenient to describe the motion in terms of how many times the object goes around in one second, rather than to say how much time it took to go around. For example, let's say a ball goes around a circle with a period of 0.2 seconds. In one second, it would go around 5 times. The quantity 5 per second is called the frequency, f.
Period, T and frequency f are related by

$$Frequency = \frac{1}{Period}$$

$$f = \frac{1}{T}$$

Units: Period has units of time (sec). Frequency has units of 1/sec

While frequency is usually defined with units of 1/s, a related unit is revolutions per minute or rpm. rpm tells you how many times the object goes around the circle in *one minute*. This unit is frequently used for gears, motors, and in industry settings.

Example 2.

A boy runs laps on a circular track of radius 28 m. He runs with a constant speed of 4 m/s.
(a) Calculate the period
(b) Calculate the frequency
(c) Draw a motion diagram.
(d) Draw a diagram of his path, and show where he will be after 33 s.

Solution:

r = 28 m; v = 4 m/s;

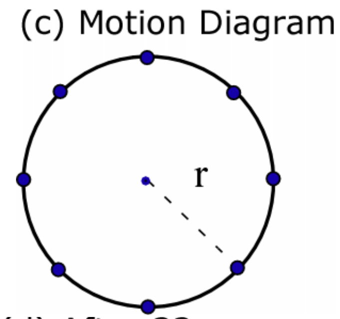
(a) T = ?
The period of his motion is 44 s.

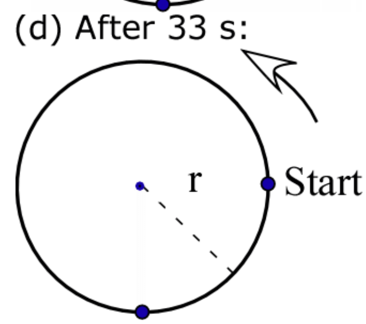
$$v = \frac{2\pi r}{T}$$
$$T = \frac{2\pi r}{v}$$
$$= \frac{(2)(3.1416)(28m)}{4m/s} = 44s$$

(b) f = ?

$$f = \frac{1}{T} = \frac{1}{44} = 0.023/s$$

The frequency of his motion is 0.023/s.



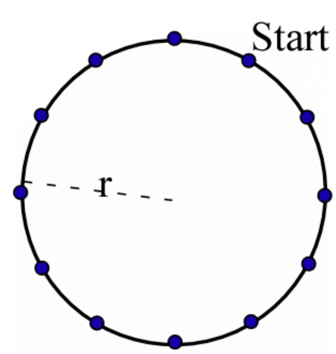


(d) Since it takes him 44 s to go one full circle around the track, after 33s he will be $\frac{33}{44} = \frac{3}{4}$, or three quarters of the way around the track. If he travels in a counterclockwise (CCW) direction, he will be at the position shown in the diagram. (Note: we must indicate whether he goes in a CCW or CW direction!)

Example 3.

On the right is the motion diagram of a light attached to the rim of a wheel in a circus display. The wheel has a radius of r = 3.6 m, and each time click is 0.02 s.
(a) Calculate the period
(b) Calculate the frequency
(c) Calculate the velocity.

Solution:
(a) If each time click is 0.2 s, there are 12 time clicks in the circle (count!), so the period is 0.02x12 = 0.24 s



(b) f = ?

$$f = \frac{1}{T} = \frac{1}{0.24} = 4.17/s$$

The frequency of his motion is 4.17/s.

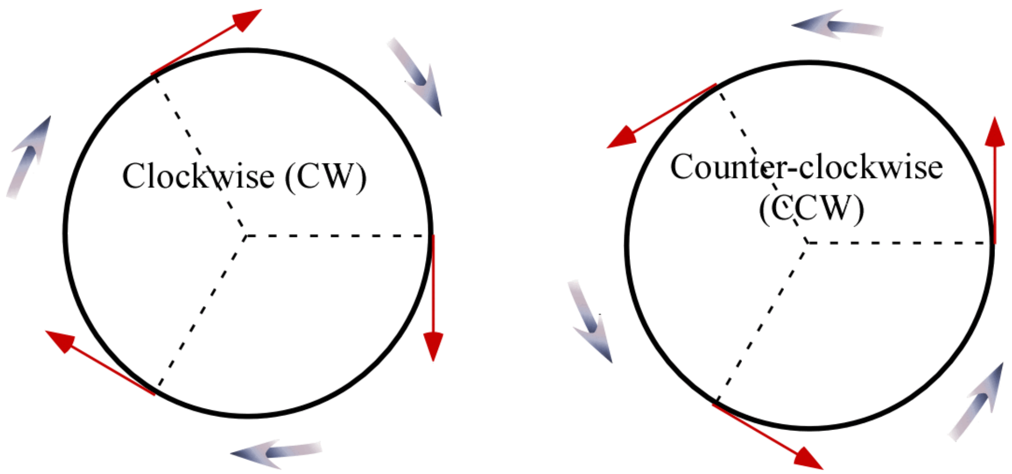
(c) v = ?

$$v = \frac{2\pi r}{T} = \frac{(2)(3.1416)(3.6m)}{0.24s} = 94.3m/s$$

His speed is 94.3 m/s.

The tangential velocity

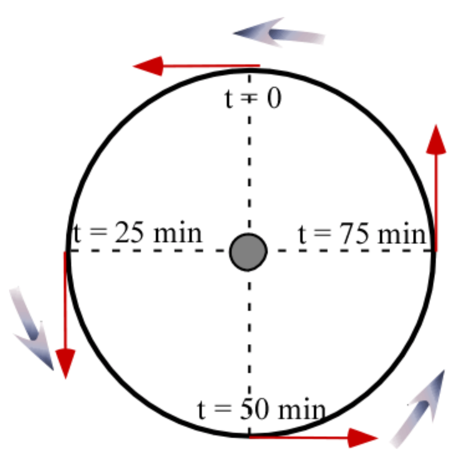
The velocity is just the speed with the direction defined. At every point along the object's path the direction of the velocity changes while the magnitude (speed) remains the same. The object can travel in a clockwise direction (CW) or counterclockwise direction (CCW). The velocity vector at any instant in time shows the direction of motion of the object.



Example 4:

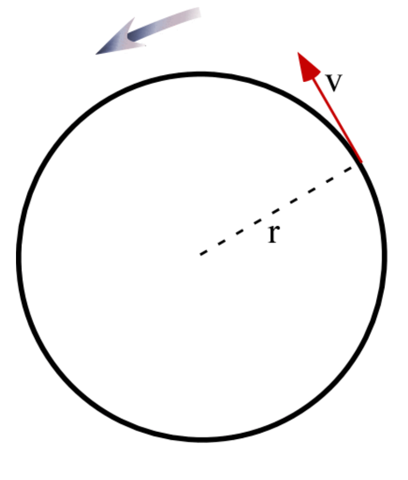
A spaceship circles the earth in a CCW path every 100 min. Draw its velocity at 4 points along its path, at t = 0, t = 25 min, t = 50 min, and t = 75 min. (Choose an arbitrary point for t = 0).

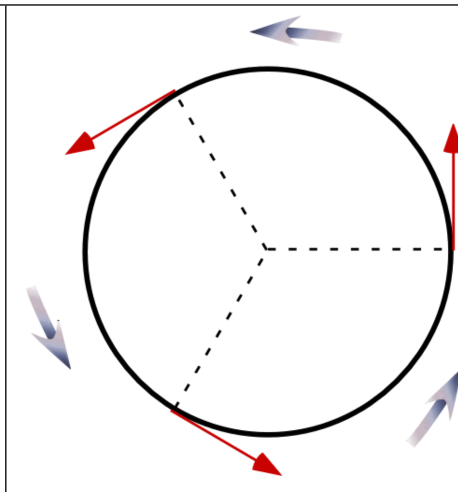
Solution:

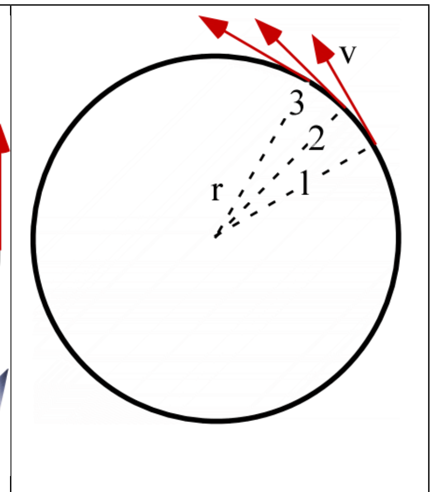


Reading Page – Forces in Circular Motion

When an object travels in a circular path, the direction of its velocity vector changes continually.

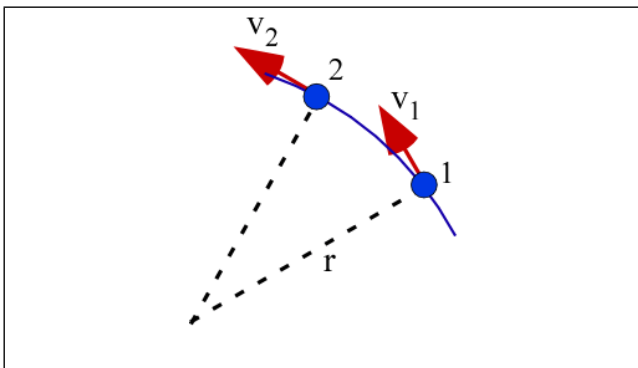


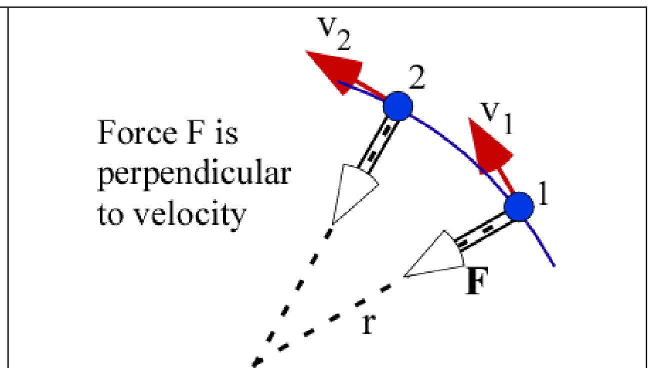




The velocity of an object in a circular path is shown by the v-arrow. This velocity is along the tangent to the circle, called the *tangential velocity*.	As the object travels in the circular path, the direction of its tangential velocity continually changes, as shown at three points along its path are shown.	If we look at three points that are close together 1, 2, and 3), the change in the direction of the tangential velocity is more obvious.

Since velocity is a vector, it has both a magnitude (which we call speed) and a direction. If either the magnitude or the direction (or both) change, the object experiences an acceleration. Since the speed does not change in uniform circular motion, only the *direction* of the velocity changes. Of course, where there is an acceleration, there must be a force!





In this expanded view, we can see that the velocity vector at 1 needs to change direction – its tip must rotate downward – to become the velocity vector at 2.	If a force F is applied to an object at position 1 the velocity vector will change its direction.

As you have seen in the Forces of Circular Motion Lab, the force that keeps things moving along a circular path is along the radius of the circle. This force is perpendicular to the velocity, which is along a tangent to the circle. This force can arise from a variety of forces, such as the tension in the string as in the Forces of Circular Motion Lab. For a car traveling along a curved road, friction provides this force. For planets, or an airplane turning, it is gravity. In all case, the force that keeps objects moving in a circle is directed toward the center of the circle. Therefore it is given the general name of *centripetal force*.

Note that the centripetal force is not a "real" force – it is a net force caused by other forces such as gravity, tension, friction or other force. The centripetal force, regardless of its origin, is related to the object's mass, its speed and the radius of its path.

(Note: you may remember from geometry that the tangent and the radius of a circle are always perpendicular to one another.)

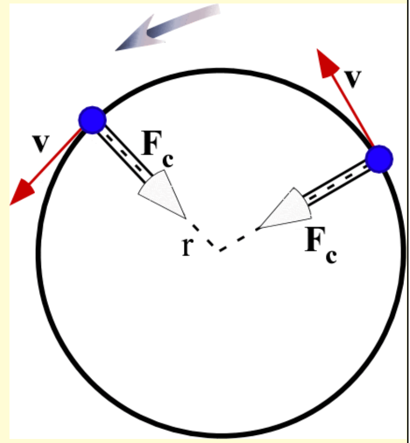
The centripetal force is affected by the following factors:

- The *mass of the object, m*
- Its *speed, v*
- The *radius of the circular path, r*

The mathematical formula for the centripetal force is

$$F_c = \frac{mv^2}{r}$$

At any given point in the circular path, the object experiences a centripetal force *along the direction of the radius at that point*.



From the formula, notice that as the *radius increases*, the *centripetal force decreases*. Thus taking a sharp turn (small radius) in a car needs a larger centripetal force than a wide turn (large radius). A right turn needs a smaller turn radius than a left turn – and you may have experienced this force as you travel in a vehicle (especially if the driver took both turns at the same speed).

Speed enters the centripetal force as a squared term. Therefore, for the same mass and radius, a speed of 100 km/h produces *not twice but four times* the centripetal force as a speed of 50 km/h. You might have noticed that bends in a highway have lower speed limits than the speed on a straight stretch – this is the reason why!

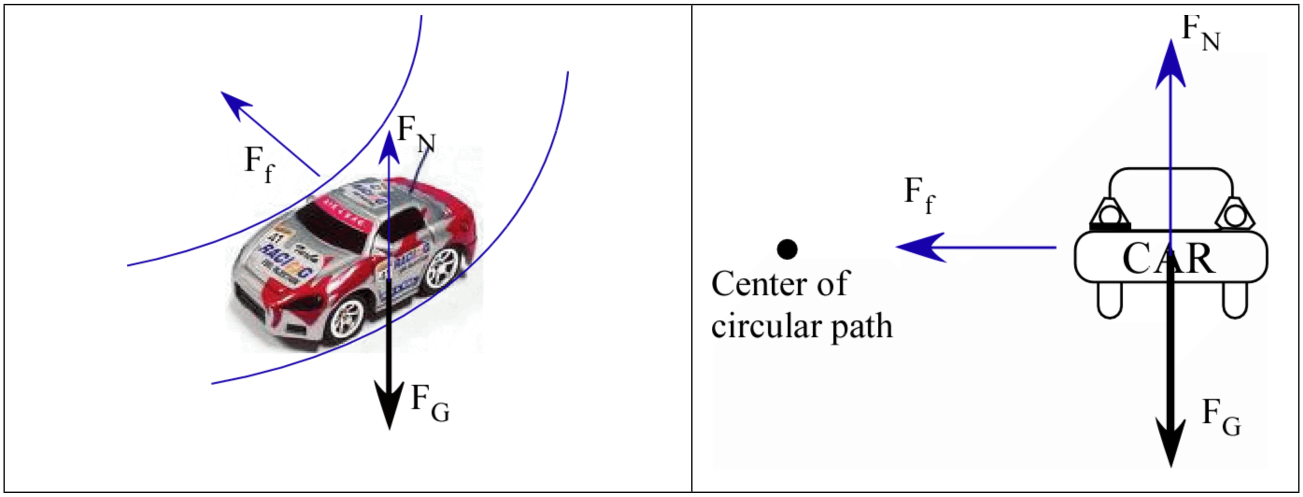
As the mass increases, the centripetal force increases. Makes sense, since the centripetal force, like any other force, can be written as "mass x acceleration." The term $\frac{v^2}{r}$ is called the centripetal acceleration.

What causes centripetal forces?

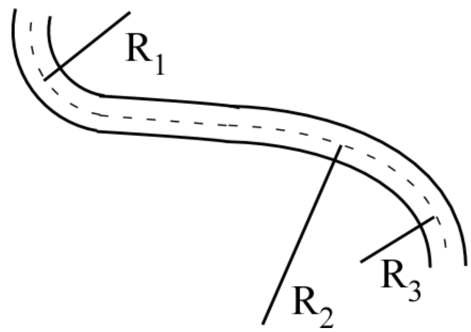
A centripetal force is different from other forces in that it is a *net force* that points toward the center of the circular path. It is caused by other forces, so it is a not a separate force, such as gravity or tension. In the case of a ball attached to a string that is swirled around in a circular path, the tension in the string is the centripetal force. When the moon orbits around the earth, gravity is the centripetal force. For a car traveling on a curved path, friction is the centripetal force – the friction that keeps the car from sliding sideways.

Shown below is a car that travels on a curve in a road. Even though the curve is only a quarter of circle, the car experiences uniform circular motion and therefore centripetal forces while on the curve. What allows the car to make the turn? Well, let's think of what would prevent it from turning. If the road were very icy and therefore frictionless, the driver would not be able to make the car turn. The car would just keep sliding in a straight line because of its inertia. Therefore, it must be friction that allows it to turn at all. In fact it is friction pushing *sideways* on the tire – and therefore toward the center of the circle. We know friction must push sideways because we need a sideways force to make the velocity vector change direction!

The free-body diagram on the car looks as shown below – in two different views.



Centripetal force occurs whenever there is a curved path – it does not have to be a full circle. For example, bends in road may have several curves of different radii. The centripetal force at that point in the road will vary with its radius. For example, in the diagram shown, the radius R_3 is the smallest, R_1 is medium and R_2 is the largest. The centripetal force is the most for R_3 and the least for R_2 (for a vehicle traveling at the same speed).



The actual way the force is produced depends on the particular situation:

Situation	The centripetal force is produced by
Planetary orbits	gravitation
Electron orbits	electrostatic force on electron
Centrifuge	contact force (reaction) at the walls
Gramophone needle	the normal force on walls of the groove in the record
Car turning a corner	friction between road and tires
Car turning a corner on a banked track	component of normal force
Aircraft banking	horizontal component of lift on the wings

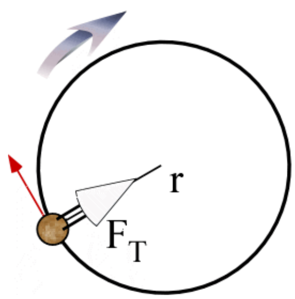
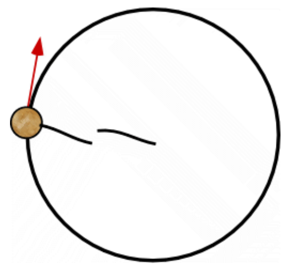
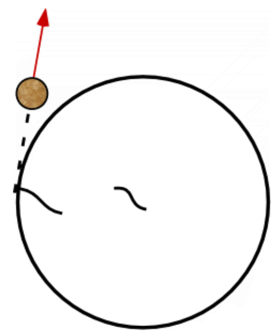
So a centripetal force may be a contact force, electrostatic, magnetic, gravitational, etc.

An interesting example is a helium-filled balloon inside a cornering car. The balloon leans in towards the center of the circle. The air in the car tries to continue in a straight line, so it is slewing to the right inside the car. The balloon is lighter than the air, so it gets pushed towards the lower pressure at the center of the circle.

(This section modified from http://www.ioppublishing.com/activity/education/Teaching_Resources/Teaching%20Advanced%20Physics/Mechanics/Circular%20motion/page_3850.html)

Consequences of tangential velocity

Since the velocity of an object on a circular path is always at a tangent, what do you think will happen when the centripetal force "suddenly" disappears? For example, imagine that a ball is being swung in a horizontal circle, and the string breaks suddenly. Well, the centripetal force (and acceleration) disappear, and the ball's velocity won't change any more - which means that it will continue traveling in the direction of the velocity just as the string broke.

		
1. Ball is swung in a circular path, held by a taut string. The string's tension produces the centripetal force.	2. The string breaks	3. The ball travels in the direction of the velocity vector at the moment it broke (when there are no other forces).

Centripetal or Centrifugal?

You will probably hear people saying "centrifugal" rather than "centripetal" force, and describing it as an *outward* force when an object is traveling in a circle. After all, you do feel pushed up against the door of the car when you take a turn, or you push outward when you turn your bicycle. The so-called centrifugal force is just a fictitious force that you feel because you are in a moving system.

Fictitious? Isn't science fact, not fiction? Well, when one is in a moving system you think you feel forces, but they are really just a consequence of inertia. For example, if you are riding a bus, and the driver suddenly brakes, you feel "thrown forward" – as if a force pushed you forward. In reality, your body wanted to continue at the same velocity as before (due to inertia), but the bus slowed down and got in your way. Similarly, when you are in a car and it goes around a curve, your body wants to travel in a straight line because of inertia, but the door of the car turns and gets in your way – and you get the distinct feeling of being pushed up against it!

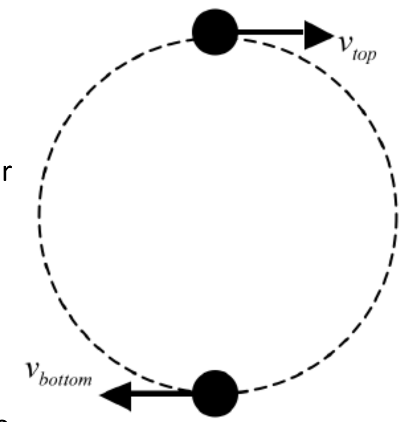
So centrifugal forces are fictitious from that standpoint. From the point of view of a person standing on the curve and watching the bus or the car, all they saw was inertia in action, and the centripetal force when the car took a turn.

Centripetal Acceleration (optional)

Although the labs make an empirical case for the centripetal acceleration being $a = \frac{v^2}{r}$, how or why this particular dependence on velocity and radius comes about appear a bit mysterious. A relatively simple description (although it is not exact) can be made in order to show its dependence on velocity and radius.

Consider the velocities exactly at the top and bottom of the trajectory so that they lie along the same line (although pointing in opposite directions). This allows us to calculate a linear acceleration that should be representative of the centripetal acceleration. It takes a half-period in time to go from the top to the bottom. The acceleration is the change in velocity per change in time:

$a = \frac{v_{top} - v_{bottom}}{T/2} = 4\frac{v}{T}$ where v is the speed and T is the period.



Therefore, the acceleration increases as either the speed increases or the period decreases. But it is important to remember that the period *also* depends on the speed: $T = \frac{2\pi r}{v}$. We can see that the period decreases with larger speed so the effect of increasing speed has two ways to increase the acceleration: one through the velocity per time relation, but another occurs through decreasing the period. The period also gets smaller with radius, which will increase the acceleration. We can put the expression for the period into the above acceleration to get $a = \frac{2}{\pi}\frac{v^2}{r}$, which is almost correct except for the numerical factor of $\frac{2}{\pi}$. A more rigorous derivation would not have this factor. But the current discussion gives some insight as to why the centripetal acceleration has its particular dependence on velocity and radius.

Reading Page: Defining Linear Momentum

Defining Linear Momentum

Momentum is a commonly used term in everyday life. A basketball rolling on the floor has momentum (is on the move) and is going to take some effort to stop. A bowling ball rolling on the floor has a lot of momentum (is really on the move) and is going to be harder to stop. Momentum (specifically linear momentum) is a physics term; it refers to the quantity of motion that an object has. If an object is in motion ("on the move") then it has momentum. Momentum can be defined as "mass in motion." All objects have mass; so if an object is moving, then it also has momentum - it is mass in motion. If the object is not moving, it still has momentum but its momentum is zero. The amount of momentum which an object has is dependent upon two variables: how much stuff is moving and how fast the stuff is moving. Momentum depends upon the variables mass and velocity. The linear momentum of an object is equal to the mass of the object times its velocity. Because velocity is a vector, the linear momentum is also a vector. Linear momentum is denoted by the letter p and for simplicity, from now on, "linear momentum" will be referred to as simply "momentum".

The momentum $\vec{p}$ of an object of mass m moving with a velocity $\vec{v}$ is defined as the product of the mass m and the velocity $\vec{v}$ of the object.

$$\vec{p} = m\vec{v}$$

Momentum is a vectorial quantity that has the same direction as the velocity of the object and a magnitude m times the velocity. The standard unit for measuring momentum is

$$[p] = [m][v]$$

$$[p] = \text{kg} \bullet \frac{\text{m}}{\text{s}}$$

From the definition of momentum, one can see that a heavy object will have a larger momentum than a light object moving at the same speed. Also, objects with the same mass have different momentum if their speeds are different.

Calculating the momentum of an object

Example 1:

A ball of mass 0.5 kg starts rolling on the ground with a speed of 5 m/s.

a) Calculate the momentum of the ball when it starts rolling.

b) After 10 seconds, the speed of the ball is 2 m/s. What is the momentum of the ball now?

Solution:

a) When the ball starts moving, the momentum of the ball is:

$$p = mv \Rightarrow p = (0.5)(5)$$

$$p = 2.5\ \text{kg} \times \frac{\text{m}}{\text{s}}$$

b) After 10 seconds, the speed of the ball changed, therefore its momentum changed too.

$$p = mv \Rightarrow p = (0.5)(2)$$

$$p = 1 \text{ kg} \times \frac{\text{m}}{\text{s}}$$

Calculating the momentum of a system of particles/objects

We often study the motion of more than one particle or object. When the system of interest is made up of several objects, the total momentum of the system is calculated as the sum of the momenta of the particles that makes up the system.

$$\overrightarrow{p_{total}} = \overrightarrow{p_1} + \overrightarrow{p_2} + \overrightarrow{p_3} + \ldots$$

Since momentum is a vector, the direction of the momentum of each object must be considered when calculating the total momentum.

Example 2:

Paul is playing with two identical balls, each of mass 0.2 kg. He throws the first ball up with an initial speed of 5 m/s and the second ball down with an initial speed of 5 m/s.

a) Calculate the momentum of the first ball just when it is thrown and specify its direction.

b) Calculate the momentum of the second ball just when it is thrown and specify its direction.

c) Calculate the total momentum of the system made up of the two balls.

Solution:

a) The momentum of the first ball that is thrown up will be oriented in the same direction as the velocity of the ball: up.

Given: $m_1 = 0.2 \text{ kg}$ $v_1 = 5 \text{ m/s}$ Unknown: $p_1 = ?$	$p_1 = m_1 v_1$ $p_1 = (0.2)(10)$ $p_1 = 2 \text{ kg} \times \frac{\text{m}}{\text{s}}$ oriented vertically up.

b) The momentum of the second ball thrown down, will have the same direction as the velocity of the ball: down.

c) The total momentum of the system made up of the two balls is the vectorial sum of the two momenta. Because they have different directions, one must choose a positive direction first. We will consider the positive direction to be vertically up. That means that the momentum of the second ball is negative.

$$\overrightarrow{p_{total}} = \overrightarrow{p_1} + \overrightarrow{p_2}$$
$$p_{total} = p_1 + (-p_2)$$
$$p_{total} = 2 + (-2)$$
$$p_{total} = 0\ \text{kg} \times \frac{\text{m}}{\text{s}}$$

This is an interesting result. Even though each object in the system has a momentum that is not zero, the *total momentum* of the system is zero. This happens because the momentum is a vector and directions must be taken into account when adding vectors.

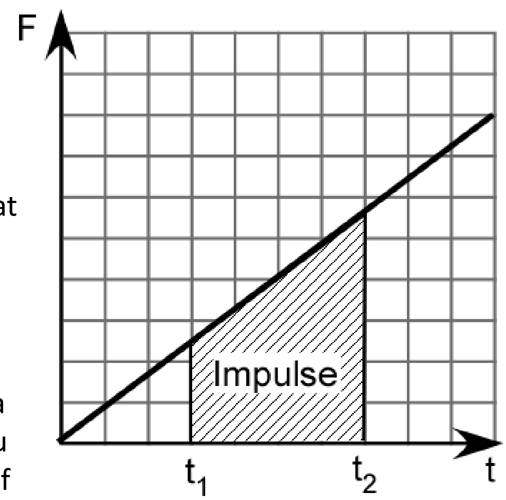
Conservation of Linear Momentum and its Relationship to Force

(optional reading page)

In the "Exploring a Collision Lab", we discovered that a collision depends on the mass as well as the velocity. It turns out that our description of Newton's law left out an important fact: Force is not mass times the rate of change of velocity (acceleration); rather it is the rate of change of momentum:

$$\vec{F} = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t} \xrightarrow{\text{correct form of Newton's Law}} \frac{\Delta\left(m\vec{v}\right)}{\Delta t}$$

where $\vec{p} = m\vec{v}$ is called the **linear momentum**.

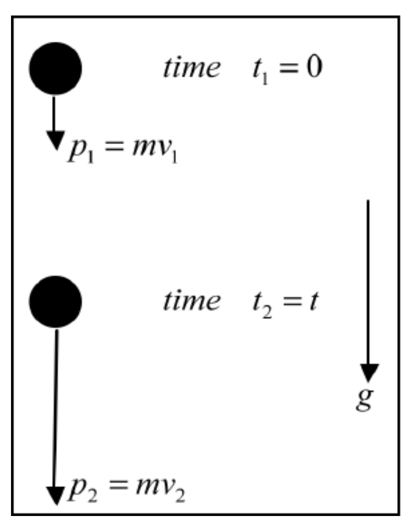


Sometimes physics problems are more easily understood if we consider the system over some time interval, rather than looking at the rates of change due to a force applied at some instant of time. The *area* under a Force-time plot is called the **impulse** and it is related to the change of momentum over that occurs over the time interval: $\Delta\vec{p} = \vec{F}\,\Delta t$, where $\Delta t = t_2 - t_1$, is obtained directly from Newton's law. This says that if you apply a force over some time interval, you will get a corresponding change in momentum. One example where impulse is widely used is for rating rocket engines which apply a thrust (a force) for some period of time. Model rocket engines that you purchase at a hobby shop are rated by their impulse in units of Newton-seconds.

Example 1. To see that the physics works out the same way as our earlier usage of Newton's laws, we will re-examine a ball falling under a gravitational force.

The force of gravity, mg, acts over a time interval of $\Delta t = t_2 - t_1$ so that the change in momentum is given by

$$\begin{aligned}\Delta p &= p_2 - p_1 = mv_2 - mv_1 = m\left(v_2 - v_1\right)\\ &= F\,\Delta t = \left(-mg\right)\left(t - 0\right) = -mgt\end{aligned}$$

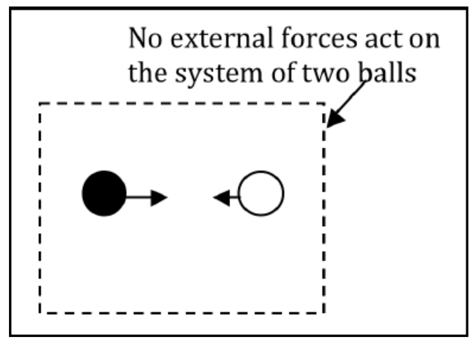


This gives the change in velocity $v_2 - v_1 = -gt$, which is the same result we obtained earlier for an object falling in gravity.

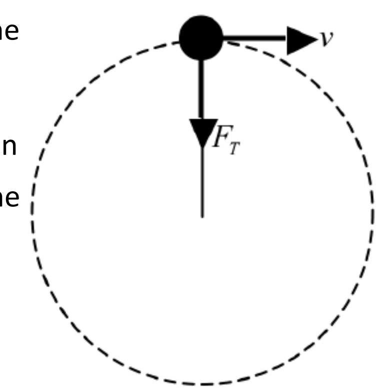
Conservation of Linear Momentum:

A very useful application of the momentum construction occurs when there is no force acting during the time interval Δt, being considered. If the force is zero, then the momentum does not change over that time interval: $\Delta\vec{p} = 0$. This is known as *conservation of linear momentum*. Another way to say this is if we know the momentum at one time, then the momentum will be the same at any time later if no forces act during that time interval. This can be a very powerful idea, as shown in the next example which illustrates a collision.

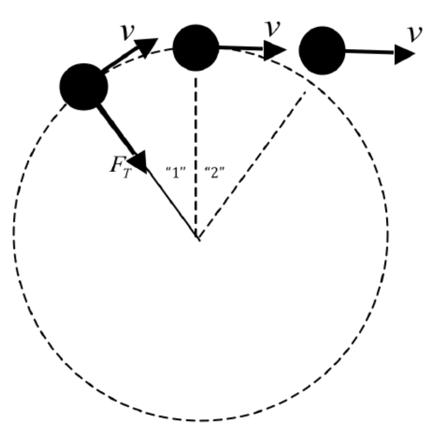
Example 2. Consider two balls, each having a momentum a given at time t_1 just before the balls collide. At this time, the dark ball has moment p_1^{dark} and the light ball has momentum p_1^{light}. If we consider the two balls as one "system" where no forces external to that system act on the two balls, then we know that their *total* momentum will be the same before and after (at time t_2) the collision: $p_1^{dark} + p_1^{light} = p_2^{dark} + p_2^{light}$. The beauty of this is that we can ignore the forces (which could be very large!) during the collision because they are *internal* to the system. In fact, there are many internal forces at work. For example there are molecular forces that are holding the material together in each of the balls. But if we consider the total momentum of the two balls, then their total momentum is the same before and after their collision as long as there are no external forces acting on them. (For example, example 1 showed that the momentum would change if gravity was acting).



Example 3. It can be difficult to explain to students what the trajectory of a ball undergoing circular motion on a string will be when you cut the string. The difficulty arises if you use F=ma which requires you to consider the force at a particular *instant* of time. If the string is cut at the instant shown in the following snap-shot of the ball, will the tension force F_T cause the ball to move downward rather than horizontal in the next
instant of time (not shown in the diagram)?



By using momentum conservation and impulse, we can more clearly show how this works by considering very *small* time intervals before and after the string is cut. In time interval "1", the impulse from the tension force F_T acting over this time interval causes a change in momentum
so that the velocity changes its direction (not its speed) to bring the ball to the top position shown in the diagram. If the string is cut at the instant the ball is at the top position, then during the entire time interval "2" there is no force acting on the ball. By the conservation of linear momentum just discussed, we can say that there will be no momentum change during this second time interval (or any time interval thereafter).



Reading Page: Change in Momentum and Conservation of Linear Momentum

Calculating the change in momentum

Up until now, in all the above examples, we calculated the momentum of an object or system but there was no collision. What happens with the momentum of an object in a collision? Well, if we consider the simple case of a ball falling toward the ground, colliding with the ground and then moving upward, then one thing that we know is that the velocities of the ball before and after collision are not the same. Even if the ball rebounds with the same speed, after it hits the ground, the direction of the velocity is different: downward before, upward after it rebounds. Therefore, there will be a change in the momentum of the ball.

To calculate the change in the momentum of an object (or system) due to a collision, one must calculate the momentum immediately before the collision and immediately after collision. The change in momentum will be:

$$\Delta \vec{p} = \overrightarrow{p_{after}} - \overrightarrow{p_{before}}$$

Example 1:

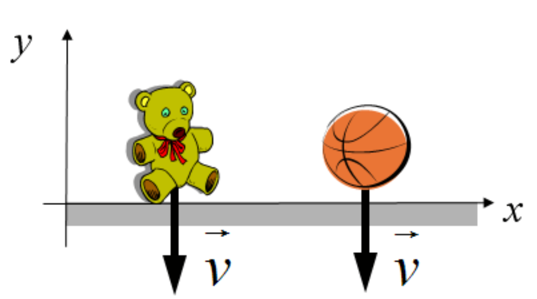
A plush bear and a ball, both of mass m, are falling toward the ground. Just before colliding with the ground, both have the same speed, v. After the collision, the bear does not move anymore. The ball rebounds with the same speed as before the collision.

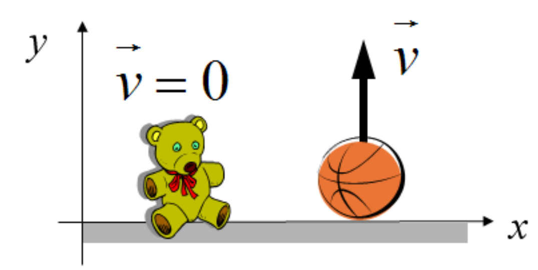
a) Find the change in the momentum of the bear.

b) Find the change in the momentum for the ball.

Solution:

Below you have a schematic drawing of what happens to the bear and ball immediately before and after collision with the ground.

<table>
<tr>
<td>Before collision:


Known:
$m_{bear} = m_{ball} = m$
$v_{bear} = v_{ball} = v$</td>
<td>After collision:


Known:
$v_{bear} = 0$
$v_{ball} = v$</td>
</tr>
</table>

a) Before the collision, the momentum for the bear is $p_{before} = mv$. After the collision, the bear does not move, therefore its speed is zero and thus its momentum after the collision is zero too, $p_{after} = 0$. Because momentum is a vector, when calculating the change in momentum one must define a positive direction: consider the positive direction to be vertically up. In that case, for the bear, the change in momentum is:

$$\Delta \vec{p} = \overrightarrow{p_{after}} - \overrightarrow{p_{before}}$$

Consider the positive direction to be vertically up.

$$\Delta p_{bear} = p_{after} - p_{before}$$

$$\Delta p_{bear} = 0 - (-mv) = mv$$

Don't forget that the velocity of the bear before collision is oriented down (in the negative direction of the vertical axis), therefore the minus sign in the parenthesis.

b) Before collision, the momentum of the ball is $p_{before} = mv$ oriented the same way as the velocity of the ball, downward. After the collision, the momentum of the ball is $p_{after} = mv$ oriented the same way as the velocity of the ball, upward. Considering the positive direction to be vertically up, the change in momentum for the ball is:

$$\Delta \overrightarrow{p_{ball}} = \overrightarrow{p_{after}} - \overrightarrow{p_{before}}$$

$$\Delta p_{ball} = p_{after} - \left(-p_{before}\right)$$

$$\Delta p_{ball} = mv - (-mv) = 2mv$$

It is worthwhile remembering that the change in momentum for the ball (rebounding object) is twice as large as the change in momentum for the bear (object sticks after collision).

The Law of Conservation of Momentum

The reason we introduce momentum for an object or system is because there is something special about it. During collisions the momentum of a system remains constant. That means that the *total momentum of an isolated system immediately before a collision is equal to the total momentum of the system immediately after the collision*. This statement is called the law of conservation of momentum. This law can also be stated in a different way: the change in the total momentum of a system due to a collision is zero.

Let's consider for example the case of a two cars colliding: Car A is moving to the right and car B is moving to the left. They collide and stick together. After collision they move as one single object. If we calculate the total momentum of the system (the two cars) immediately before collision and immediately after collision, we will find out that the total momentum of the system has the same value.

The opposite of a collision is an interaction that forces two objects apart. These interactions are called *explosions*, even though they may lack a flash or a pop. As an example, imagine that you are standing still on ice skates, and you throw a heavy ball directly forward. Before you throw the ball, you and the ball form a closed system with total momentum of zero because neither one is moving. After you throw the ball, both you and the ball move but in opposite directions. Now imagine that one of your friends videotapes this, and runs the movie backwards. What do you see? You see two objects (you and the ball) moving toward each other and then colliding. This is what we call a collision. When you run the movie forward though, you see two objects that were together and then move apart. This is what we usually call an explosion. But in physics, an explosion is nothing else than a collision run backwards.

Let us consider a different example. A rifle is suspended by two thin strings in a horizontal position. The rifle and the bullet inside the rifle constitute the two objects that make up the system. Before the explosion (firing the rifle), both the rifle and the bullet have no velocity therefore the momentum of each is zero (and the total momentum of the rifle + bullet system is zero) before the explosion. After the explosion (the rifle is fired), the bullet is propelled forward with a very high speed and the rifle is propelled backward with a much smaller speed.

The conservation of momentum law states that the total momentum of the system must be the same before and after the explosion (collision). Therefore, the total momentum of the rifle + bullet system must be zero after the explosion because it is zero before the explosion. Does this make sense? Let's write out the momentum of the system after the explosion. Because the rifle and the bullet move in opposite direction, we must choose one direction as the positive direction of motion: the direction in which the rifle moves will be considered the positive direction.

$$\overrightarrow{p_{system}} = \overrightarrow{p_{rifle}} + \overrightarrow{p_{bullet}}$$
$$p_{system} = p_{rifle} + \left(-p_{bullet}\right)$$
$$p_{system} = m_r v_r + (-m_b v_b)$$
$$p_{system} = 0 \Rightarrow m_r v_r = m_b v_b$$

So what do we learn from this?

First, that momentum, like velocity, is a quantity that is described by both magnitude and direction; we measure both "how much" and "which direction." Momentum is a vector. Therefore, when momenta act in the same direction, they are simply added; when they act in opposite directions, they are subtracted.

Second, that during a collision/explosion, only the *total* momentum of the system is conserved and not the momentum of each individual object in the system. The momentum of the rifle or the bullet before explosion is zero. After explosion neither is zero because both the bullet and the rifle are moving! Therefore the momentum of each *individual* object is NOT conserved during collisions/explosions. But the total momentum of the system is conserved: for the system of rifle and bullet, no momentum was gained, none was lost.

Third, that if a bullet moves with a high speed after firing a gun, then the gun's recoil speed is small if the mass of the gun is large.

Depending on what happens to the objects after a collision, we can classify the collisions into two different categories:

Inelastic collisions: after collision the objects that collide stick together. An explosion is also an inelastic collision that runs in reverse. Examples of inelastic collisions are: two cars colliding and moving together after collision, a fish eating another fish, a bullet fired into a piece of wood, a rifle firing a bullet (explosion), a running child jumping on a stationary skateboard, a child on a moving skateboard jumping off the skateboard, two stationary ice skaters pushing off each other, etc.

Elastic collisions: after collision the objects that collide continue to move independent of each other. Examples of elastic collisions are: two cars colliding and continuing to move independently, a bowling ball colliding with a pin, a tennis ball colliding with a racquet, a football colliding with the foot of a player, a moving car colliding with a stationary car and moving independently after collision, etc.

Regardless of the type of collision that takes place, the law of conservation of momentum applies to all collisions.

For collisions one is usually interested in finding the speed of the colliding objects after the collision if the speed of those objects before the collision is known. Using the law of conservation of momentum allows one to determine the speeds after collision or the speeds before collision if the speeds after collision are known.

Problem Solving Strategies for Collision Problems:

Step 1: Clearly define the system (how many objects make up the system). If possible, choose a system that is isolated $(\vec{F}_{net} = \vec{0})$ or within which the interactions/collisions are sufficiently short and intense that you can ignore external forces for the duration of the interaction. In such interactions/collisions the total momentum of the system is conserved.

Step 2: Visualize the problem. Make a schematic drawing of the before and after collision for the system. On that figure (or separately) define symbols that will be used in the problem, list known values, and identify what you're trying to find.

Step 3: Calculate the total momentum of the system before collision, after collision and make the two equal. This means applying the law of conservation of momentum:

$$\overrightarrow{p_{systemBC}} = \overrightarrow{p_{systemAC}}$$

$$\overrightarrow{p_{1BC}} + \overrightarrow{p_{2BC}} + \overrightarrow{p_{3BC}} + ... = \overrightarrow{p_{1AC}} + \overrightarrow{p_{2AC}} + \overrightarrow{p_{3AC}} + ...$$

Step 4: Check that your result has the correct units, is reasonable, and answers the question.

Example 2:

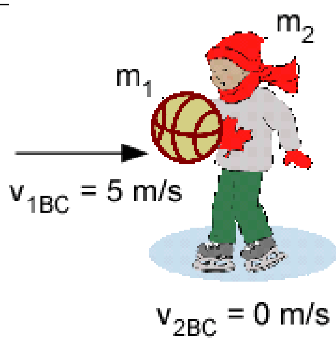
Joe is standing still in the middle of an ice rink. Timi throws him a medicine ball and Joe catches it. Just before Joe catches the ball, the ball is moving with a speed of 5 m/s. Knowing that Joe's mass is 55 kg and the ball's mass is 5 kg, find the speed with which Joe (holding the ball) starts sliding on the ice.

Solution:

Step 1: The system is made up of two objects: the ball and the Joe. The two objects undergo an inelastic collision; after colliding, the two objects move together with the same speed.

Step 2: A schematic drawing of the system before and after collision is shown below.

Before collision:



Known:

$m_1 = 5\text{ kg} \qquad v_{1Before} = 5\text{ m/s}$

$m_2 = 55\text{ kg} \qquad v_{2Before} = 0\text{ m/s}$

After collision:

Known:

$m_1 + m_2 = 60\text{ kg}$

Unknown:

$v_{After} = ?\text{ m/s}$

Step 3: The total momentum of the system is conserved.

The total momentum of the system before collision is:

$$\vec{p}_{sysBefore} = \vec{p}_{ballBefore} + \vec{p}_{boyBefore}$$
$$p_{sysBefore} = p_1 + p_2$$
$$p_{sysBefore} = m_1 v_1 + m_2 v_2$$
$$p_{sysBefore} = (5)(5) + (55)(0)$$
$$p_{sysBefore} = 25\text{ kg} \times \frac{\text{m}}{\text{s}}$$

The total momentum of the system after collision is:

$$\vec{p}_{sysAfter} = \vec{p}_{ballAfter} + \vec{p}_{boyAfter}$$
$$p_{sysAfter} = p_1 + p_2$$
$$p_{sysAfter} = m_1 v_{After} + m_2 v_{After} = (m_1 + m_2) v_{After}$$
$$p_{sysAfter} = (60) v_{After}$$

But because the total momentum is conserved, we have:

$$\vec{p}_{sysBefore} = \vec{p}_{sysAfter}$$
$$p_{sysBefore} = p_{sysAfter}$$
$$25 = 60(v_{After})$$
$$v_{After} = \frac{25}{60}$$
$$v_{After} = 0.41\text{ m/s}$$

Step 4: Joe holding the ball is moving slowly with a speed of 0.41 m/s in the *same* direction as the direction of motion of the ball. Notice that before and after velocities are both positive.

Example 3:

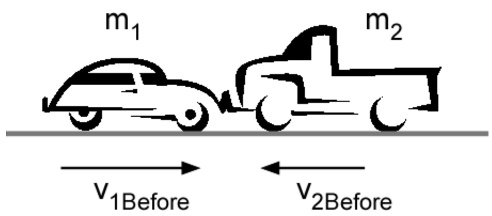
A small car and a truck move toward each other. The car (mass = 750 kg) moves at a speed of 25 m/s and the truck (mass = 1200 kg) moves at a speed of 20 m/s. They collide, lock bumpers together, and move together after the collision. What is their speed after collision, and in which direction are they moving?

Solution:

Step 1: The system is made up of two objects: the car and the truck. The two objects undergo an inelastic collision.

Step 2: A schematic drawing of the system before and after collision is shown below.

Before Collision:

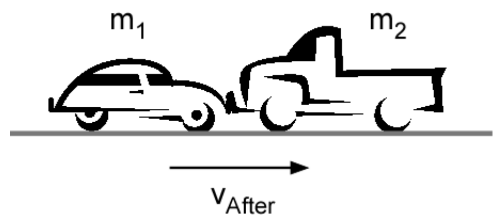


Known:

$m_1 = 750\text{ kg} \quad v_{1Before} = 25\text{ m/s}$

$m_2 = 1200\text{ kg} \quad v_{2Before} = 20\text{ m/s}$

After Collision:



Known:

$m_1 = 750\text{ kg}$

$m_2 = 1200\text{ kg}$

Unknown:

$v_{After} = ?\text{ m/s}$

Step 3: Because the car and truck move in opposite directions, let's choose the right to be the positive direction of motion. After the two objects collide and move together, we do not know the direction which they will move. Let's assume that both cars move to the right. The calculation will give us both the sign (direction) and magnitude of v_{After}. During the collision the total momentum of the system is conserved.

Let's calculate the total momentum of the system before collision:

$$\overrightarrow{p_{sysBefore}} = \overrightarrow{p_{carBefore}} + \overrightarrow{p_{truckBefore}}$$

$$p_{sysBefore} = p_1 + (-p_2) = p_1 - p_2$$

$$p_{sysBefore} = m_1v_1 - m_2v_2$$

$$p_{sysBefore} = (750)(25) - (1200)(20)$$

$$p_{sysBefore} = 18750 - 24000$$

$$p_{sysBefore} = -5250\text{ kg} \times \frac{\text{m}}{\text{s}}$$

The total momentum of the system before collision is to the left (in the negative direction). Because the total momentum is conserved, the momentum of the system after collision must also be oriented to the left.

After collision, the total momentum of the system is:

$$\overrightarrow{p_{sysAfter}} = \overrightarrow{p_{carAfter}} + \overrightarrow{p_{truckAfter}}$$

$$p_{sysAfter} = p_1 + p_2$$

$$p_{sysAfter} = m_1v_{After} + m_2v_{After} = (m_1 + m_2)v_{After}$$

$$p_{sysAfter} = (750 + 1200)(v_{After})$$

$$p_{sysAfter} = 1950(v_{After})$$

It does not matter that we obtained a positive value for the momentum after collision: when calculating the velocity we will eventually get a negative value for it (see below).

From the conservation of momentum we have:

$$\overrightarrow{p_{sysBefore}} = \overrightarrow{p_{sysAfter}}$$

$$p_{sysBefore} = p_{sysAfter}$$

$$-5250 = 1950\left(v_{After}\right)$$

$$v_{After} = -\frac{5250}{1950}$$

$$v_{After} = -2.69 \text{ m/s}$$

Step 4: The result shows that the velocity of the system (two cars locked together) is oriented toward the left (in the negative direction).

Example 4:

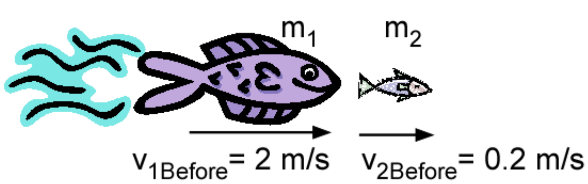
A big fish of mass 3.4 kg is swimming at a speed of 2 m/s when suddenly swallows a smaller fish of mass 0.3 kg swimming in the same direction in front of him at a speed of 0.2 m/s. What is the speed of the big fish immediately after his feast?

Solution:

Step 1: The system is made up of two objects: the big fish (#1) and the small fish (#2). Big fish swallowing small fish is equivalent to two objects colliding and moving together after collision, which is an inelastic collision.

Step 2: Visualize the collision.

Let's start by making a schematic drawing of the situation *Before Collision* (swallowing):



Known:

$m_1 = 3.4 \text{ kg} \quad v_1 = 2 \text{ m/s}$

$m_2 = 0.3 \text{ kg} \quad v_2 = 0.2 \text{ m/s}$

After Collision (the big fish swallows the small fish), the system is still made up of two objects except that one object (small fish) is inside the big fish.

Known:

$m_1 + m_2 = 3.4 + 0.3 = 3.7 \text{ kg}$

Unknown:

$v_{1\,After} = v_{2\,After} = v_{After} = ? \text{ m/s}$

Step 3: Both objects are moving in the same direction, therefore the direction of motion of the big fish will be considered the positive direction (to the right as in the drawing). During collisions the total momentum of a system is conserved.

Let's calculate the momentum of the system before the collision (swallowing):	After collision (swallowing), the total momentum of the system is:
$\overrightarrow{p_{sysBefore}} = \overrightarrow{p_{BigFishBefore}} + \overrightarrow{p_{SmallFishBefore}}$ $p_{sysBefore} = p_1 + p_2$ $p_{sysBefore} = m_1 v_1 + m_2 v_2$ $p_{sysBefore} = (3.4)(2) + (0.3)(0.2)$ $p_{sysBefore} = 6.8 + 0.06$ $p_{sysBefore} = 6.86 \text{ kg} \times \frac{\text{m}}{\text{s}}$	$\overrightarrow{p_{sysAfter}} = \overrightarrow{p_{BigFishAfter}} + \overrightarrow{p_{SmallFishAfter}}$ $p_{sysAfter} = p_1 + p_2$ $p_{sysAfter} = m_1 v_{After} + m_2 v_{After} = (m_1 + m_2) v_{After}$ $p_{sysAfter} = (3.4 + 0.3)(v_{After})$ $p_{sysAfter} = 3.7(v_{After})$

The total momentum of the system is conserved:

$$\overrightarrow{p_{sysBefore}} = \overrightarrow{p_{sysAfter}}$$

$$6.86 = 3.7(v_{After})$$

$$v_{After} = \frac{6.86}{3.7}$$

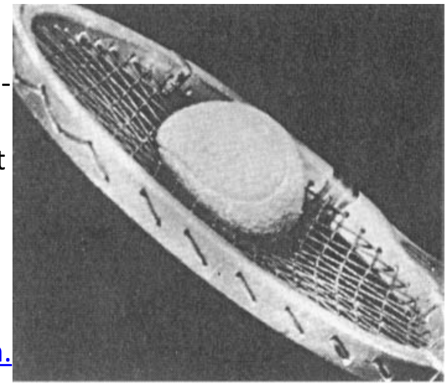
$$v_{After} = 1.85 \text{ m/s}$$

Step 4: The big fish is moving with a smaller speed after swallowing the small fish, its mass is larger.

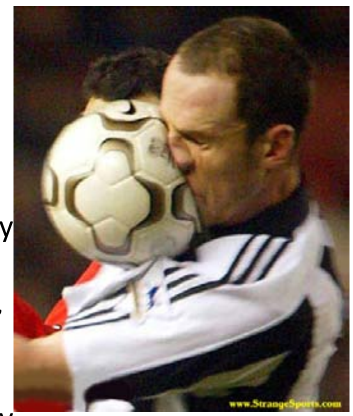
Reading Page: Force and Momentum

Powerpoint: Crumple Zones.ppt

How does momentum relate to force?



A collision is a short-time interaction between two objects. For example, the collision between a racquet ball and a wall, or a tennis ball and a racquet, or a baseball and a bat, may seem instantaneous to the eye. A slow-motion look at the tennis ball/racquet collision in the picture below reveals that the net of the racquet is deformed, and the ball itself may have a slight deformation. It takes time to distort the net of the racquet, and more time for the ball to re-expand as it leaves the net. (Picture from http://yhspatriot.yorktown.arlington.k12.va.us/~dwaldron/p_classroom.html)



The same can be said about a soccer ball hitting you in the face (see picture below, from www.strangesports.com/images/content/106516.jpg). This is a vivid example of a collision. During such a collision (or any collision), the two objects (ball and head) exert forces on each other according to Newton's third law: you can definitely feel that force on your face and the ball deforms due to that force.These forces are not constant. They vary in a very complex way during the time of the collision. Due to these forces, the velocity of the ball as well as the velocity of the head changes, and therefore the momentum of the objects that collide changes. Using Newton's second law to predict the outcome of such a collision is very difficult because the forces acting during collisions are not constant. Many times though we are interested in evaluating forces exerted on objects during collisions.

Since forces act during collisions and the momenta of objects change during collisions, let's try and look for a connection between them. Let's consider the following simple example: a ball is thrown against a wall. The ball hits the wall with a velocity v_{Before}. There will be an interaction force between the ball and the wall and as a result, the momentum of the ball will change. This interaction force is not constant and varies during the time of the collision. The ball leaves the wall (rebounds) with a velocity v_{After}.

From Newton's second law, we can calculate the average interaction force acting on the ball during collision:

$$\left.\begin{aligned} F_{avg} &= ma_{avg} \\ a_{avg} &= \frac{\Delta v}{\Delta t} = \frac{v_{AC} - v_{BC}}{\Delta t} \end{aligned}\right\} \Rightarrow F_{avg} = m\frac{v_{AC} - v_{BC}}{\Delta t}$$

$$F_{avg} = \frac{m\left(v_{AC} - v_{BC}\right)}{\Delta t}$$

$$F_{avg}\Delta t = m\left(v_{AC} - v_{BC}\right)$$

To truly understand the equation, it is important to understand its meaning in words. In words, it could be said that the force times the time it acts equals the mass times the change in velocity. In physics, the quantity $F_{avg} \cdot \Delta t$ is known as *impulse*.

The impulse can be expressed in a different way also: $F_{avg}\Delta t = mv_{AC} - mv_{BC}$

The quantity mv_{BC} represents the momentum of the ball before collision and the quantity mv_{AC} represents the momentum of the ball after collision. Therefore, $mv_{AC} - mv_{BC} = \Delta p$, represents the change in momentum for the ball. Thus the above equation really states that $F_{avg}\Delta t = \Delta p$

Impulse = Change in momentum

To summarize, in a collision, an object experiences a force for a specific amount of time which results in a change in momentum (the object's mass either speeds up or slows down). The impulse experienced by the object equals the change in the object's momentum. The impulse is equal to the collision force times the time for which this force acts.

For the same change in momentum, $mv_{AC} - mv_{BC} = \Delta p$, one may have larger or smaller forces acting during the collision. It all depends on the *time* of the collision. If one wants to reduce the amount of force in collisions, one must increase the time interval for the collision, for the same impulse applied (change in momentum). Below are several examples where increasing the time of the collision serves to reduce the impact force on the object.

Example 1: Cars and Air Bags

In a car accident, two cars can either collide and bounce off each other or collide and crumple together and travel together with the same speed after the collision. But which type of collision is more damaging to the occupants of the automobiles - the rebounding of the cars or the crumpling up of the cars? Contrary to popular opinion, the crumpling up of cars is the safest type of car collision. If cars rebound upon collision, the momentum change will be larger and so will the impulse. A greater impulse means a larger force. Smaller forces on people inside cars decrease injuries. Therefore cars are now made with crumple zones, which are sections in cars that are designed to crumple up when the car is in a collision. Crumple zones minimize the effect of the force in an automobile collision in two ways. The crumpling of the car lengthens the time over which the car's momentum is changed; by increasing the time of the collision, the force of the collision is greatly reduced. Also by crumpling, the car is less likely to rebound upon impact, thus minimizing the momentum change and the impulse.

How do air bags in cars help in case of collisions? The force of interaction between a person and an inflated air bag is reduced because the time it takes the person to stop is increased, the air bag has to deflate. The slower the bag deflates, the longer it takes for the momentum of the person to change, therefore the smaller the force applied

Example 2: Bumper cars

(Picture from http://media.merchantcircle.com/

Bumper car rides are designed so that the cars can collide without much danger to the riders. Each car has a large rubber bumper all around it. This rubber bumper prolongs the time of impact and thus reduces the force of the collision when the momentum of the bumper car must change from a certain value to zero (from moving, the car comes to rest).

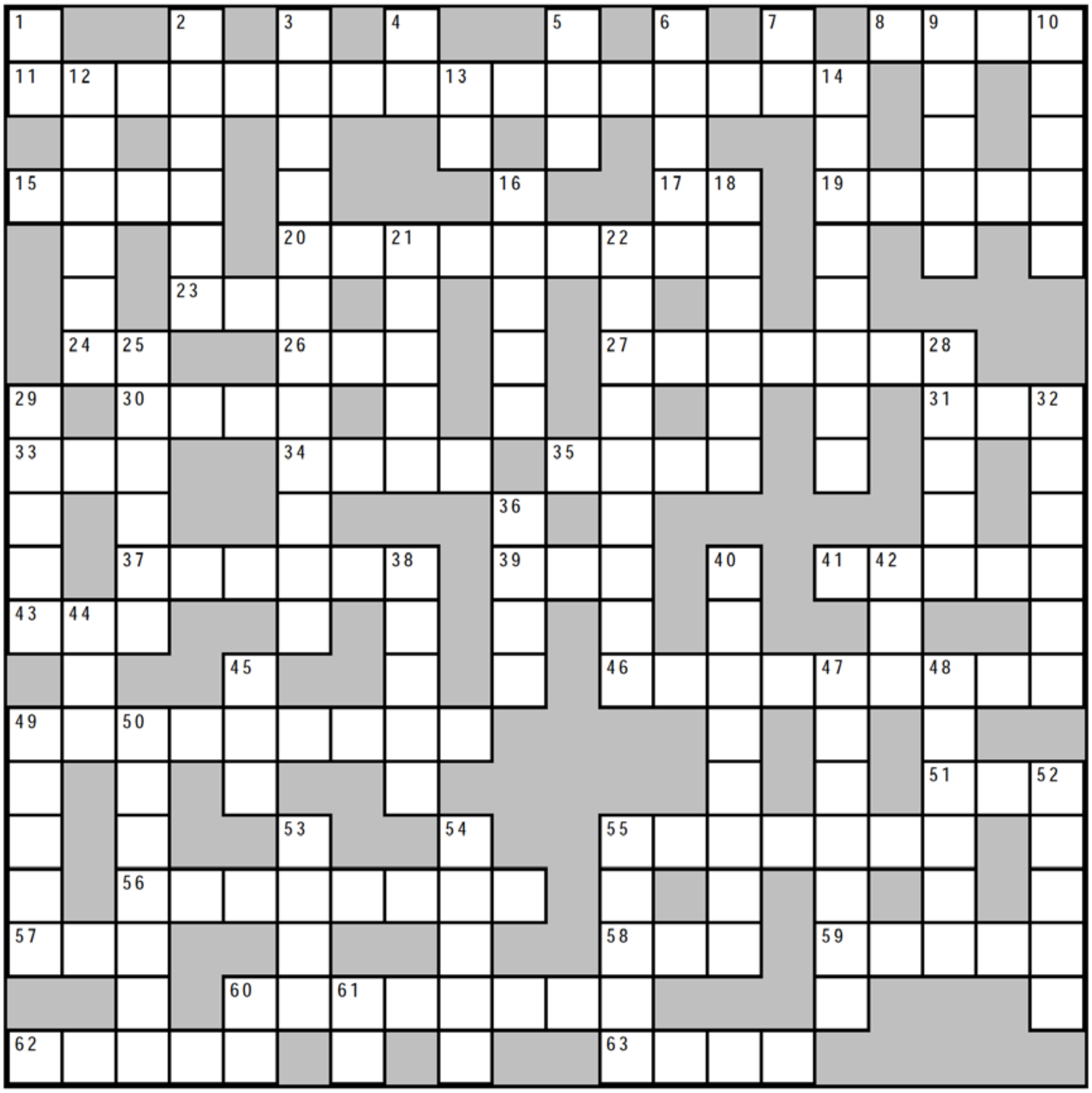
Example 3: Brick Breaking

(Picture from heavyfists.com)

How can people break bricks with just a stroke? It is not about how strong the person is as much as how fast the person hits the brick! The idea of short time of contact explains how a karate expert can sever a stack of bricks with the blow of his bare hand. He brings his arm and hand swiftly against the bricks with considerable speed (momentum). This momentum is quickly reduced when he delivers an impulse (force) to the bricks. The impulse is the force of his hand against the bricks multiplied by the time his hand makes contact with the bricks. By swift execution he makes the time of contact very brief and correspondingly makes the force of impact huge. If he makes his hand bounce upon impact (move backwards), the force is even greater because the change in momentum will also be greater (see the bear and ball example). If one wants to apply a big force one must make the time of interaction with the brick very short.

Physics Crossword Puzzle

(from the Book of Phyz by Dean Baird)



CLUES

ACROSS

8 One billionth prefix

11 Giving human characteristics to inanimate objects

15 What the ugly duckling became

17 Calculator manufacturer, not Casio

19 Momentum is measured in kilogram-_?_s per second

20 When a single body is forcefully divided into two or more parts

23 Van Gogh died with very little money and only one _?_

24 Equal to a kg·m/s

26 A military academy on the eastern seaboard

27 A type of collision in which kinetic energy is conserved

30 1,000,000,000,000 prefix

31 Unit of electrical resistance

33 79% N_2, 19% O_2

34 Impact force is inversely proportional to impact _?_

35 A word that implies division by time

37 F = ma is not the way Newton revealed his_?_ law in the *Principia*

39 Consume edible matter

41 Italian nuclear physicist Enrico; 10^{-15} m

43 Deux en Español

46 An event in which two bodies undergo an abrupt interaction

49 Is credited with developing the principle of conservation of momentum

51 Greek letter; "British" unit of pressure

55 HCl, NaCl, H_20, CO_2, and $C_6H_{12}O_6$ are chemical _?_s

56 Momentum is conserved in any system in which no _?_ forces act

57 Chimpanzee, gorilla, orangutan, _?_

58 The result of unbalanced, external force

59 Eastern Indian stringed musical instrument that relies on resonance

60 One piece of the result of an explosion

62 The kinds of perfect conditions under which introductory physics occurs

63 The answer to #34 across spelled backwards

DOWN

1 Bay State; F =

2

3 A principle that applies to a quantity that does not change

4 Colorful _?_; blue ion element

5 Joule is the MKS unit of energy, _?_ is the cgs unit of energy

6 _?_gate timer

7 One one-thousandth of a second

9 Angle measuring smaller than $\pi/2$ radians

10 Sopranos, tenors, orchestra, elaborate costumes and classic story lines; small window on a car

12 $F = \Delta p/\Delta t$ is _?_'s second law

13 p =

14 The product of mass and velocity; inertia in motion

16 The rate of change in momentum

18 Half of the respiration process

21 Used to separate white light into colors

22 A sticky collision

25 Force/area in solids

28 Cyan, yellow, and magenta are primary _?_s in printing

29 *Book of Phyz* author

32 Momentum was regarded by Newton as "quantity of _?_"

36 10^{15} prefix

38 1980's Barry Levinson feature film about life in Baltimore in the early 1960's

40 $(N\cdot s)/(m/s)$

42 Developers and administrators of the SAT, SAT II, and AP Exams, among others

44 The tangent of $\pi/4$ rad

45 A hypothesis that has been verified through extensive testing over a long period of time

47 Change in momentum

48 If you increase the _?_ time, you decrease the _?_ force

49 The name of the Greek letter commonly used to denote change

50 1970's/80's UK band that enjoyed hits with "Tempted" and "Pulling Mussles (From the Shell)"

52 The chemical nature of the noble gases

53 2000 was a leap _?_, but 1700, 1800, and 1900 were not

54 Molten constituent of the Earth's mantle

55 The modern version of this woodwind is commonly made of metal

60 The Sunshine State

61 Yellowish precious metal

Applications of Newton's Laws: Review

Free Fall

Free fall refers to objects falling under the influence of gravity.

Acceleration a = g = -9.8 m/s^2, where the negative sign indicates that the acceleration is downward.

Parameters and equations that apply to free fall:	
v_i is the initial vertical velocity v_f is the final vertical velocity Δt is the time a_y is the vertical acceleration (usually = g)	Equations of Motion: Equation # 1: $v_f = v_i + a\,\Delta t$ Equation # 2: $\Delta y = v_i \Delta t + \frac{1}{2} a(\Delta t)^2$ Equation # 3: $v_f^2 - v_i^2 = 2a\Delta y$

Projectile Motion

Projectile motion refers to motion that has horizontal and vertical components. The horizontal component is determined by the horizontal velocity, and has no acceleration, a_x=0. The vertical motion is determined by gravity, just as in free fall.

Parameters and equations that apply to two-dimensional trajectories:	
Horizontal motion:	Vertical motion:
v_{ix} is the initial horizontal velocity v_{fx} is the final horizontal velocity a_x is the horizontal acceleration (usually = 0) Δt is the time, which is the same for both horizontal and for vertical motion	v_{iy} is the initial vertical velocity v_{fy} is the final vertical velocity a_y is the vertical acceleration (usually = g)
$v_{fx} = v_{ix}$ (since $a_x = 0$) $\Delta x = v_{ix} \Delta t$	$v_{fy} = v_{iy} + a_y \Delta t$ $\Delta y = v_{iy}\Delta t + \frac{1}{2} a_y (\Delta t)^2$ Since $a_y = g = -9.8 m/s^2$ in SI units, $v_{fy} = v_{iy} + (-9.8)\Delta t$ $\Delta y = v_{iy}\Delta t + \frac{1}{2}(-9.8)(\Delta t)^2$

Circular Motion

An object that moves with a *constant speed in a circular path* is said to have uniform circular motion. Uniform, because its speed does not change; circular because it travels in a circle of fixed radius (r).

Parameters and Equations that apply to circular motion	
T = period of motion (the time the object takes to complete one complete circle)	Speed of object undergoing uniform circular motion: $v = 2\pi r/T$
f = frequency, the number of cycles per second	Frequency: $f = 1/T$
v = speed, or magnitude of the tangential velocity	Centripetal force: $F = mv^2/r$
r = radius of circle	

Momentum

Momentum is defined as mass x velocity. The total momentum is conserved in all collisions.

Parameters and Equations that apply to momentum	
m_1 *and* m_2 are the masses of the colliding objects	Momentum: $p = mv$
v_{1i} and v_{2i} are the initial velocities of m_1 and m_2	Change in momentum: Δp
v_{1f} and v_{2f} are the final velocities of m_1 and m_2	Conservation of momentum: $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$
Δt = time over which collision occurs	Impulse: $F\Delta t = \Delta p$
F = average force transferred by collision	

