## Framing Questions

Note: Unless it is specified otherwise, air resistance and friction can be neglected.

1. Bond the Skydiver

James Bond is in free fall after jumping from an airplane. He sees the bad guy below, also in free fall below him. Intent on catching him, Bond flips over, stretches his arms over his head, pointing downward, and zooms downward. Do you think Bond can catch up with the bad guy? Explain your reasoning.

## 2. Acceleration at the top

What will be the acceleration of a ball kicked straight upward at the moment it reaches the very top of its trajectory?


## 3. Two Golfers

Andy and Jan decide to hit golf balls off a cliff onto a flat pasture below. Both hit the balls horizontally, and at the same time. Andy hits his ball so it travels at a larger velocity than Jan's. Whose ball hits the ground first? Whose ball hits the ground farther? Explain your reasoning.


## 4. Two Golfers, again

The second time around, Andy hits his ball so it travels a little upward while Jan hits her ball so it travels horizontally. Both balls have the same speed as they leave their respective tees. Whose ball hits the ground first?

5. Cut the swinging ball

A ball is tied to end of a string, and allowed to swing. When it reaches the very bottom, the string is cut just above the ball with a sharp razor blade.

Which path will the ball follow: A, B or C? Explain your reasoning.

6. Ball in a tube

A ball is shot into the center of a spiral tube that is lying on a table (neglect gravity). Which path will it follow when it emerges? Explain.

7. Suppose an open railroad car is rolling without friction in a vertically falling downpour and an appreciable amount of rain falls into the car and accumulates there. Consider the effect of the accumulating rain on the speed and momentum of the car.
a) The speed of the car will
[A] increase
[B] decrease
[C] not change
b) The momentum of the car will
[A] increase

[B] decrease
[C] not change
8. A patio table is protected by an umbrella.
a) Which situation causes a larger force on the open umbrella?
[A] Rain falling on the umbrella
[B] Hail falling on the umbrella?
[C] Light snow falling on the umbrella
b) Which is more likely to damage the umbrella?
[A] Rain

[B] Hail
[C] Snow
9. A one pound lump of clay traveling at one foot per second smashes into another one pound lump of clay which is not moving. Smush! They stick and become one two pound lump. What is the speed of the two pound lump (neglect friction)? Explain.


## Practice 5.1: Acceleration from Velocity vs. Time Graphs

1. A spark timer is set to 10 Hz . This setting creates a spark 10 times a second, or one spark every 0.10 seconds. This means that the time between sparks is 0.10 seconds.

A record is made on spark timer tape of a car accelerating across the table. The position of the dots is presented in the data table below.
a) By just looking at the numbers (no calculation), circle the words that describe the motion of the car: Speeding up OR Slowing down ?

Positive Velocity OR Negative Velocity ?
b) Use this data and obtain a velocity-time graph for the car. You may have to add columns to the table.

| Time (s) | Position of <br> dots (cm) |
| :---: | :---: |
| 0 | 0.0 |
| 0.10 | 0.1 |
| 0.20 | 0.5 |
| 0.30 | 1.1 |
| 0.40 | 1.9 |
| 0.50 | 3.0 |
| 0.60 | 4.3 |
| 0.70 | 5.9 |
| 0.80 | 7.7 |
| 0.90 | 9.7 |
| 1.00 | 12.0 |


c) Plot a velocity vs. time graph (above).

Use this graph to describe the motion of the car as it moves across the table.
d) Draw a motion diagram of this motion. How does it compare to the spots on the spark tape?
e) From the velocity vs. time graph determine the acceleration of the car as it moves across the table.
2. On a fictitious distant planet Amethyst, an astronaut drops a rock from a tower, while her friend records its position every 0.2 s using a video camera. Here is the picture they got. The first spot is taken to be $\mathrm{t}=0$, where its velocity is $\mathrm{v}=0$.

Position marked every 0.2 s

|  |  |  |
| :--- | :--- | :--- |
|  |  |  |

a) Read the diagram and create a data table (use blank table).


b) Draw a position-time graph (above).
c) Calculate the average velocity during each time interval, and add velocity-time data to the data table (above).
d) Draw a velocity-time graph (below).

e) From the v-t graph, calculate the acceleration on the planet.
3. A v-t graph is shown for free fall of an object on a planet somewhere in the universe. Calculate the acceleration due to gravity on that planet.

4. Two objects fall, and their v-t graphs are shown in the diagram. One of them experiences more drag due to air resistance than the other.
a) Just by looking at the graphs, explain whether
A or B speeds up at a faster rate.
b) Calculate A's acceleration.

c) Calculate B's acceleration.

## Practice 5.2: Conceptual Problems on Acceleration and Gravity

1. Analyze the velocity and draw motion diagrams for the toy car below:

Picture | How does the starting velocity (v) change when the car |
| :--- |
| undergoes acceleration a as shown |
| Description: |
| Motion diagram: |

2. Describe how the ball's starting velocity (v) changes when it experiences the acceleration shown. Your description should address both the direction and the magnitude of velocity.

| Picture | Description | Motion Diagram |
| :---: | :---: | :---: |
| a) |  |  |
| b) |  |  |
| c) |  |  |
| d) |  |  |

3. Compare the motion of balls $A$ and $B$. The initial velocities $(v)$ are different, but the accelerations are the same for $A$ and $B$.

4. Compare the motion of balls $C$ and $D$. The initial velocity $(v)$ is the same for $C$ and $D$, but the accelerations are different (the balls are on two different planets).

5. A ball is dropped off a cliff. Draw a motion diagram. Draw arrows to indicate the velocity and acceleration.
6. A ball is thrown off a cliff with a downward velocity of $3 \mathrm{~m} / \mathrm{s}$. Draw a motion diagram. Draw arrows to indicate the velocity and acceleration. Compare this motion diagram to the one in the previous problem (problem 5).
7. A ball is thrown upward with a velocity of $12 \mathrm{~m} / \mathrm{s}$. Draw a motion diagram, indicating its motion every 0.2 sec . Draw arrows to indicate the velocity and acceleration. Compare this motion diagram to the one in problem 5.
8. The two graphs on the $y$-t graph show the same ball dropped on two different planets.
a) Which planet makes the ball accelerate more? Explain.

9. A $v$-t graph is shown in the center diagram below.

a) Describe a situation that would create such a v-t graph.
b) Draw the $y$-t and a-t graphs for this situation.

## Practice 5.3: Velocity and " g "

10. If a rock is dropped on earth with a starting velocity of $v_{i}=0$, what will its velocity be in the next few seconds?
(a) Use the table below to write the values. Assume that the acceleration due to gravity is $9.8 \mathrm{~m} / \mathrm{s}^{2}$.
(b) Plot the velocity vs. time (on the graph below or on cm graph paper)

| $t$ (sec) | $v(\mathrm{~m} / \mathrm{s})$ |
| :--- | :--- |
| 0 | 0 |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |


11. Suppose that the same rock was dropped on the moon, where $g=-1.6 \mathrm{~m} / \mathrm{s}^{2}$. (a) Generate a similar table as in problem 1
(b) Draw the $v$ - $t$ graph on the graph in problem 1 above. Label the Earth line "Earth" and the moon line "moon."

| $t(\mathrm{sec})$ | $v(\mathrm{~m} / \mathrm{s})$ |
| :--- | :--- |
| 0 | 0 |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |

12. If Jerry drops an earth rock on earth from a height of 2 m and his astronaut brother Steve drops a moon rock from the same height on the moon, which one (if either) would take longer to hit the ground? Explain your reasoning.
13. Ariana has the unusual ability to travel to different planets/satellites/stars and play games on the surface there. If she tosses a ball upward with a velocity of $20 \mathrm{~m} / \mathrm{s}$, calculate, on each planet/ satellite/star:

|  | on Earth | on Moon | on Sun | on Mars | on Jupiter | on Pluto |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| The acceleration <br> due to gravity | $9.8 \mathrm{~m} / \mathrm{s}^{2}$ | $1.6 \mathrm{~m} / \mathrm{s}^{2}$ | $273.4 \mathrm{~m} / \mathrm{s}^{2}$ | $3.7 \mathrm{~m} / \mathrm{s}^{2}$ | $25.8 \mathrm{~m} / \mathrm{s}^{2}$ | $0.6 \mathrm{~m} / \mathrm{s}^{2}$ |
| Acceleration of <br> ball on way up |  |  |  |  |  |  |
| Acceleration at <br> highest point |  |  |  |  |  |  |
| Acceleration on <br> the way down |  |  |  |  |  |  |
| The velocity at the <br> highest point |  |  |  |  |  |  |
| The time taken to <br> get to the highest <br> point |  |  |  |  |  |  |
| The highest point <br> reached |  |  |  |  |  |  |
| The velocity when <br> the ball reaches <br> her hand again* |  |  |  |  |  |  |

[^0]14. Add your observations to the Student Summary Page: Free Fall

## Practice 5.4: Quantitative Problems - Acceleration due to Gravity

In all the problems below, assume that the acceleration due to gravity is $-9.8 \mathrm{~m} / \mathrm{s}^{2}\left(-980 \mathrm{~cm} / \mathrm{s}^{2}\right)$ in the downward direction unless otherwise stated. Use consistent units.

1. A coconut falls from a tree that is 12 m high.
a) How much time does it take to hit the ground?
b) How fast is the coconut traveling when it hits the ground?

c) Draw a diagram that (qualitatively) indicates the coconut's velocity and its acceleration when it is midway down and when it is just above the ground.
2. If rain falls from a height of 2 km , how fast should it be hitting the ground (if all you did is use the motion formulae)? Do you think it is really hitting the ground with that velocity? Why or why not?
3. An apple falls from the top of the tree and takes 2 seconds to hit the ground.
a) How far did the apple fall?
b) Calculate the velocity of the apple just before it hits the ground.
4. If you threw a ball up in the air at a velocity of $1200 \mathrm{~cm} / \mathrm{s}$ (upward)
a) What will its velocity be at the highest point?
b) How much time will it take to get to the highest point?
c) How high is the highest point?
d) How much time does the ball take to come down from the highest point?
e) What is the ball's velocity when it comes back into your hand?
5. If a cannonball were shot vertically up in the air at a velocity of $20 \mathrm{~m} / \mathrm{s}$ (upward)
a) What will its velocity be at the highest point?
b) How much time will it take to get to the highest point?
c) How high is the highest point?
d) How much time does the cannonball take to come down from the highest point?

What is the cannonball's velocity when it hits the ground?
6. A fish jumps out of the water at a vertical velocity of $1.2 \mathrm{~m} / \mathrm{s}$. How much time is the fish in the air before it strikes the water again?
7. Varni drops a ball from the edge of a bridge over a stream. She sees it hit the water 1.2 seconds later. What are two different factors she can figure out with this information?
8. Twins Tim and Mariah throw two tennis balls. Tim throws his ball upward with a velocity $\mathrm{v}_{\mathrm{o}}$ while Mariah throws it downward from the same height with the same velocity $\mathrm{v}_{\mathrm{o}}$.
a) Which one hits the ground first, if either?
b) Which one hits the ground with greater velocity, if either? Explain.
9. Dr. Martin climbs to the top of the physics building to determine the height of the building. From the edge of the roof of the building he drops a book and observes that it takes 2.25 s to strike the ground below.
a) How high is the roof of the physics building?
b) Why would it be difficult to determine the height of the building if he dropped only one page out of the book?

## Additional Problems:

10. Ginnie throws a ball up.
a) She wants it to reach a height of 26 m .
b) (a) What factor determines whether the ball will reach the desired height? (b) Calculate that factor.
11. At her graduation ceremonies, Shoshana throws her cap straight up in the air, and it comes back into her hands 2.2 sec later. If you neglect air resistance, what factors can you figure out with this information?
12. A technician at the top of a 200 m tall radio tower drops her hammer.
a) How much time does it take for the hammer to strike the ground?
b) With what velocity will the hammer strike the ground?
c) How far will the hammer have fallen after 3 seconds of fall?
13. A nut falls from a tree, falls for 1.5 seconds and strikes a rock.
a) How high above the ground was the nut before it dropped?
b) What was the velocity of the nut just before it struck the rock?
14. A baseball is hit vertically upward and travels straight up to a height of 80 meters. How fast is the ball moving when it strikes the ground?
15. Make up a problem on motion under gravity where you give only one piece of information that involves velocity in the upward direction, and you ask for all the possible information that a person can calculate.
16. Make up a problem on motion under gravity where you give only one piece of information that involves distance traveled, and you ask for all the possible information that a person can calculate.

## Practice 5.5: Motion of an Object Initially Shot Upward

1. A ball is shot directly upward from a launcher placed on a high cliff with a velocity of $30 \mathrm{~m} / \mathrm{s}$. On this planet, very similar to Earth, the acceleration due to gravity is $g=-10 \mathrm{~m} / \mathrm{s}^{2}$.
2. In the table below determine the velocity and position of the ball.

| Time, t | Velocity, v | Position, y |
| :---: | :---: | :---: |
| 0 s | $30 \mathrm{~m} / \mathrm{s}$ |  |
| 2 s |  |  |
| 4 s |  |  |
| 6 s |  |  |
| 8 s |  |  |
| 10 s |  |  |
| 12 s |  |  |

a) Make a velocity vs. time graph of this motion.
b) Describe the shape of this graph. Calculate the slope of the graph.
c) What is the slope of the $v v$ s. $t$ when the velocity of the object is $0 \mathrm{~m} / \mathrm{s}$ ?

d) What is the displacement of the object when the velocity is $+10 \mathrm{~m} / \mathrm{s}$ ? $-10 \mathrm{~m} / \mathrm{s}$
e) What is the maximum displacement of the ball relative to its starting point?
f) What is the velocity of the ball when it has fallen back to the original location?

## Practice 5.6: 2D Motion: Data and Motion Diagrams

1. Zak throws a wad of paper horizontally from his balcony. His friend Melba takes a movie of it and produces the following data of its trajectory as it falls to the ground. The first data point was at the moment Zak threw the wad of paper, and the last data point was when the wad hit the ground. (Note, due to air resistance the downward acceleration might not be $9.8 \mathrm{~m} / \mathrm{s}^{2}$ ).

| Time t <br> $(\mathbf{s})$ | Horizontal <br> position $\mathbf{x}(\mathbf{m})$ | Vertical <br> position $\mathbf{y}(\mathbf{m})$ |
| :--- | :---: | :---: |
| 0 | 0 | 0 |
| 0.2 | 0.64 | -0.16 |
| 0.4 | 1.28 | -0.64 |
| 0.6 | 1.92 | -1.44 |
| 0.8 | 2.56 | -2.56 |
| 1 | 3.2 | -4 |
| 1.2 | 3.84 | -5.76 |
| 1.4 | 4.48 | -7.84 |
| 1.6 | 5.12 | -10.24 |
| 1.8 | 5.76 | -12.96 |
| 2 | 6.4 | -16 |

a) Plot an x-t graph


Draw a motion diagram of the wad's horizontal motion (draw next to the graph above)

Describe the paper wad's motion along the horizontal direction.

Use the graph to Use the graph to figure out its horizontal velocity.
b) Plot a y-t graph


Draw a motion diagram of its vertical motion.

Use the graph to describe the paper wad's motion along the vertical direction.
c) Plot a $y$-x graph.

Draw motion diagrams for the horizontal motion and for the vertical motion.
How do these motion diagrams correlate to those you drew in parts (a) and (b)?

2. From the $y$-t data in problem 1,
a) Produce a table of vertical velocity-time data (add a column to the table in problem 1).
b) Plot the vertical velocity vs. time data. .

c) Use the vertical velocity-time graph to calculate the acceleration felt by the wad of paper
3. If Zak throws a smaller wad of paper that takes 1.25 sec to hit the ground,
a) If he threw it with the same horizontal velocity (calculated in part 1a), how far can the wad travel horizontally during this time period?
b) How far along the horizontal would the wad have traveled in 1.0 sec ?

## Practice 5.7: Quantitative Problems on 2D Motion

Note: Be sure to draw diagrams to help you visualize the problems.

1. Tom, the cat, is chasing Jerry the mouse across a long cafeteria table that is 1.5 m from the ground. Jerry steps out of the way at the last second, and Tom runs off the edge with a horizontal velocity of $8 \mathrm{~m} / \mathrm{s}$.
a) How much time does it take before Tom hits the floor?
b) How far from the table does he land?
c) Draw horizontal and vertical motion diagrams for Tom, starting at the instant he begins running across the table, and ending when he hits the ground.
2. A child tries to flick a piece of popcorn off a table into a bowl on the floor. If the bowl is 50 cm away from the table and the table is 125 cm tall, what is the horizontal velocity with which the child must flick the popcorn?
3. A ping-pong ball is flicked off the edge of a table with a horizontal velocity. The motion diagrams for the ball's horizontal and vertical motion are shown in the picture below. The time interval between two successive points on the motion diagram is 0.2 sec .

a) Construct the ball's 2D trajectory from the motion diagrams.
b) Find the amount of time it took to travel the path.
c) Calculate its horizontal velocity.
4. In an Indiana Jones movie, a car is first driven on a flat road at a constant speed for 60 s , then driven off a cliff that is 180 m high. It lands in the river below, which is at a distance of 90 m , measured horizontally, from the edge of the cliff.
a) How much time did the car spend falling down? (that is, the time interval between its leaving the edge of the cliff and landing in the river).
b) What was the horizontal velocity of the car as it left the edge of the cliff?
c) Draw qualitative graphs of the car's horizontal and vertical velocities as a function of time, starting from the time the car starts and ending when it leaves the cliff. (Label the values on the time axis).
d) Draw qualitative graphs of the car's horizontal and vertical velocities as a function of time, starting from the time it leaves the cliff and ending when it hits the river. (Label the values on the time axis).


5. An angry boss sweeps stuff off his desk, which is 1.3 m high. The first object that he sweeps off in a grand, horizontal stroke is his coffee cup, which lands on the floor 2 m from the edge of his desk ( 2 m is the horizontal distance, as measured on the floor). The second object, a calculator, lands 2.5 m from the edge of his desk; and the third, his water bottle, 2.9 m from the edge of this desk.
a) Draw the trajectories for the three objects, starting from the time they leave the table and ending at the time they hit the floor.
b) Calculate the times it takes for the three objects to hit the ground.
c) Calculate their horizontal velocities.
d) Judging from the velocities, do you think there is a chance that the boss will calm down?

## Additional Problem

6. Wily Coyote is hard on the heels of the Roadrunner. Suddenly, Roadrunner runs off the edge of the cliff, horizontally. Across the canyon is a ledge, 25 meters away and 15 m down. Roadrunner was running at $12 \mathrm{~m} / \mathrm{s}$.
a) Does he land on the overhang?
b) If not, at what velocity should Roadrunner have been running to land on the overhang?
c) Draw a picture to illustrate Roadrunner's trajectory.

## Summary Page: Comparing Free Fall to Projectile Motion

| Compare: | Ball dropped from a height | Ball thrown horizontally from a height |  |
| :--- | :---: | :---: | :---: |
|  | Vertical motion | Horizontal motion | Vertical motion |
| Trajectory |  |  |  |
| Motion diagram |  |  |  |


| Compare: | Ball dropped from a height | Ball thrown horizontally from a height |  |
| :--- | :---: | :---: | :---: |
|  | Vertical motion | Horizontal motion | Vertical motion |
| Acceleration vs <br> time graph |  |  |  |

## Practice 5.8: Trajectory Challenge

1. Does a ball dropped out of the window of a moving car take longer to reach the ground than a ball dropped out of a stationary car? Explain your reasoning.
2. Draw the trajectories of a ball dropped from a moving car, and a stationary car. (You are standing by the side of the road, watching).

3. Draw trajectories of a ball thrown horizontally, and of another ball thrown at an upward angle:

4. Compare the trajectories of three balls thrown with the same velocity, but at different angles (their paths can cross in the figure below):


## Practice 5.9: Conceptual Problems on Angular Trajectories



1. A stuffed toy is launched as shown in the diagram above. Draw horizontal and vertical motion diagrams. Include position and velocity.
2. In the diagram above,
a) What is the horizontal acceleration of the toy at $x=4 m$ ? At $x=10 \mathrm{~m}$ ? At $\mathrm{x}=16 \mathrm{~m}$ ? Explain.
b) What is the vertical acceleration of the toy at $\mathrm{y}=4 \mathrm{~m}$ ? At $\mathrm{y}=10 \mathrm{~m}$ ? At $\mathrm{y}=16 \mathrm{~m}$ ? Explain.
c) Add the horizontal and vertical accelerations to the motion diagram above.
3. Four projectile launchers are placed side by side at different angles. Launcher A points along the horizontal. B points at $30^{\circ}$ to the horizontal, C at $45^{\circ}$ and D at $60^{\circ}$. They all launch balls at the same instant, with the same initial speed.

a) Draw trajectories for each of the balls (use different colors, if possible). The trajectories should be qualitatively accurate.
b) Which of the trajectories launches the ball to the highest point (along the vertical)? Explain.
c) Which of the four, if any, has a trajectory that sends the ball the farthest distance (along the horizontal)? Explain.
d) Are there any angles that send the balls the same distance (along the horizontal)? Explain.
e) Are there any angles that send the balls up the same height (along the vertical)? Explain.
4. Of the trajectories in problem 3,
a) Which one of them most closely resembles a pitched baseball? Explain.
b) Which one resembles a football? Explain.
c) Which one resembles a basketball shot from close to the basket? Explain.

## Practice 5.10: Circular Motion Problems

1. Rank in order, the period of circular motion for the five objects shown below, from smallest period to largest period.

A.

E.
smallest period $\qquad$ -- $\qquad$ largest period Explain your reasoning:
2. In which of the paths in problem 1 does the object have CW motion? CCW motion?
3. Two objects, $P$ and $Q$, travel in separate circular paths of the same radius. $P$ has a period of $2 s$, and $Q$ a period of 3 s .
a) Without knowing the radius, describe which one travels with a larger velocity around the path.
b) Draw the paths of the two objects. Both $P$ and $Q$ start at the bottom of their respective circles (mark it "start"). Mark where the objects will be after 1 sec . Explain your reasoning.
4. A constant speed car is tied with a string that makes it travel in a circular path. Its string suddenly breaks. What does the car do? Draw a diagram.

5. A ball travels in a circular path with a period of 0.2 sec .
a) How many times does it go around the circle in one second?
b) How many times does the ball in part (a) go around the circle in one minute? (in units of revolutions per min or rpm).
6. Suppose that a toy is attached to the ceiling and is swung in a circle a little below the ceiling. You observe that it travels in a path of radius 0.4 m . What measurements would you need to make to figure out its velocity?
7. A particular constant speed car runs at a speed of $0.42 \mathrm{~m} / \mathrm{s}$ in a circular path of radius 0.6 m . Suppose we changed its batteries so that it now runs at half the speed.
a) If we kept the radius of the path the same, would anything change?
b) If so, what would change, and how would it change?
8. A bear runs restlessly around the edge of his circular cage at a rate of 4 circles per minute.
a) What is his period (in sec)?
b) Draw a diagram of his motion, running in a clockwise path. Draw his tangential velocity at 4 points along his path.

## Additional Problems

9. A ball is tied to a string, and is twirled overhead in a circular path by the teacher. The class clown sneaks up behind the teacher, and cuts the string. What happens? Draw a diagram.
10. The radius of the bear's circular cage is 4 m . After the bear has had his lunch, he runs around the edge of his cage with a period of 12 s .
a) Calculate his velocity in $\mathrm{m} / \mathrm{s}$.
b) Convert his velocity to miles/hr.
11. The tire of a car has a radius of 30 cm . A mechanic puts the car up on a lift, and runs the engine. A spot on the edge of the tire travels at a speed of $50 \mathrm{~km} / \mathrm{hr}$.
a) Calculate the period of the tire, and its frequency.
b) The tire travels in a clockwise direction. Draw its tangential velocity at three points along its circular path.
12. A CD-ROM rotates CCW at a frequency of 200 rpm .
a) If its radius is 6 cm , what is its tangential velocity at the edge of the $C D$ ? Draw a diagram indicating the tangential velocity on the edge of the CD.
b) If instead, we look at a spot 4 cm from the center of the $C D$, what factors are the same? the rpm? the period? the tangential velocity? Explain. If any of these factors is different, calculate it/them.

## Practice 5.11: Centripetal Force Problems

1. Rank in order from smallest to largest, the centripetal force felt by the objects undergoing circular motion below. All objects have the same mass.

A.

B.
C.
D.

E.
smallest force $\qquad$
$\qquad$
$\qquad$ largest force

Explain your reasoning:
2. A ball is shot into a frictionless horizontal tube at $2 \mathrm{~m} / \mathrm{s}$. It goes through the tube and emerges on the left. Which line, $\mathrm{A}, \mathrm{B}$, or C best describes its direction of motion as it comes out of the tube? (see figure). Explain your reasoning.

3. Highway departments consider the radius of a curve when they set speed limits. The centripetal force on a car is 2500 N when it is traveling at a certain speed on section A of a curved road of radius R. How will the centripetal force be affected if the same car now travels with the same speed on section B of the road which has a curve with half the radius ( $\mathrm{R} / 2$ )? (See figure).

4. A bug sits halfway from the edge of a spinning $C D$. If it crawls toward the edge of the $C D$, will it feel more or less centripetal force? How much more or less?
5. A ball is thrown into the right hand side of a frictionless horizontal tube, as shown. Aaron says its path, just as it comes out of the tube, will be as shown in A because it loses energy and will travel in a circle of smaller radius. Bonnie says that it will continue along its circular path because of inertia, and fall right back into the tube, as shown in B. Carly says its direction is along C , because the tube is horizontal at the top (see figure above).

Do you agree with Aron, Bonnie or Carly? Explain your reasoning.


## Practice 5.12: Calculating Momentum

Based on the Reading Page: Defining Linear Momentum, answer the questions below.

1. The momentum of a stationary truck is $\qquad$
2. The momentum of a fast moving truck is (less/more/same as) the momentum of a slow moving truck. Both trucks have the same mass.
3. The momentum of a heavy moving truck is (less/more/same as) the momentum of a small car moving with the same speed.
4. A moving car has momentum $p$. If it moves twice as fast, its momentum will be
$\qquad$ (explain).
5. Two cars, one two times as heavy as the other, are moving down a hill at the same speed. Compare the momentum of the two cars.
6. Two cars with the same mass are traveling down a hill. One is traveling twice as fast as the other. Compare the momentum of the two cars.
7. If the velocity of a flying bird is oriented south, what is the direction of its momentum?
8. A car has a momentum of 1200 $\qquad$ (fill in the units).
9. If the positive direction of motion is to the right, and a car moves to the left, what can you say about the direction of the momentum of the car?

## Practice 5.13: Calculating Changes in Momentum

1. Abby is studying the change in momentum of a ball. She knows that the ball hits the ground with a speed of $6 \mathrm{~m} / \mathrm{s}$ and rebounds with the same speed. She concludes that the change in the momentum for the ball is zero because it had the same speed before and after collision. Is Abby correct? Justify your answer.
2. Two objects move toward the south. Object $A$ has a mass of 9 kg and moves at $17 \mathrm{~m} / \mathrm{s}$. Object B has a mass of 10 kg and moves at $15 \mathrm{~m} / \mathrm{s}$. Which object has more momentum?
3. What is the mass of an object moving at $6 \mathrm{~m} / \mathrm{s}$ whose momentum is $48 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}$ ?
4. What is the speed of a 4 kg object whose momentum is $8 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}$ ?
5. The following 3 questions refer to the figure below:

A

B

C

D
a) Which object above has more inertia, $\mathrm{A}, \mathrm{B}, \mathrm{C}$, or D ?
b) Which object above (A, B, C, or D) above would be the easiest to stop?
c) Which object above has the largest momentum?
6. A $0.20-\mathrm{kg}$ baseball is dropped and reaches a speed of $1.0 \mathrm{~m} / \mathrm{s}$ just before it hits the ground. It rebounds with a speed of $0.80 \mathrm{~m} / \mathrm{s}$.
a) What is the momentum of the ball before it hits the ground?
b) What is the momentum of the ball immediately after it rebounds?
c) What is the change in the ball's momentum?
7. A $0.150-\mathrm{kg}$ baseball is approaching the batter at $40.0 \mathrm{~m} / \mathrm{s}$. After the batter hits the ball, the ball leaves the bat at $30.0 \mathrm{~m} / \mathrm{s}$ in the direction of the pitcher.
a) What is the momentum of the ball before it hits the bat?
b) What is the momentum of the ball immediately after it rebounds?
c) What is the change in the ball's momentum?
8. Two carts move toward each other on an air track. Cart A has a mass of 450 g and moves toward the right with a speed of $0.850 \mathrm{~m} / \mathrm{s}$. Cart B has a mass of 300 g and moves toward the left with a speed of $1.12 \mathrm{~m} / \mathrm{s}$. What is the total momentum of the system?
9. Three identical balls (mass $\mathrm{m}=0.25 \mathrm{~kg}$ ) are thrown vertically up: ball A with a speed of $5 \mathrm{~m} / \mathrm{s}$, ball $B$ with a speed of $2.2 \mathrm{~m} / \mathrm{s}$ and ball $C$ with a speed of $7.5 \mathrm{~m} / \mathrm{s}$.
a) Rank the momentum of the balls from smallest to highest without calculating anything.
b) Calculate the momentum of each ball.
c) Calculate the total momentum of a system that contains all three balls.
d) Suppose that ball C is now thrown down instead of up. What is the total momentum of the system in this case?
10. A 0.3 kg volley ball is thrown vertically downward with a speed of $0.2 \mathrm{~m} / \mathrm{s}$. It takes it 0.08 s to reach the ground.
a) What is the speed of the ball just before it hits the ground?
b) What is the magnitude of its momentum just before it hits the ground?
11. 4 A ball of mass 0.2 kg is dropped from a height of 10 meters above the ground.
a) Calculate the speed of the ball before it hits the ground.
b) Calculate the momentum of the ball before it hits the ground.
c) The ball rebounds after hitting the ground and reaches a maximum height of 7 m . What was the speed of the ball immediately after hitting the ground?
d) Calculate the momentum of the ball immediately after hitting the ground.
e) Calculate the change in momentum of the ball during the collision with the ground.
12. Dennis is driving his $4000-\mathrm{kg}$ car and makes a 900 turn. His speed before the turn is $15.0 \mathrm{~m} / \mathrm{s}$ and after the turn it is $20.0 \mathrm{~m} / \mathrm{s}$. What is the magnitude of the change in the car's momentum? (Challenge problem)

## Practice 5.14: Conservation of Momentum

1. During a collision, the law of conservation of momentum applies to
[A] a single object
[B] two objects only
[C] any number of objects
2. An artillery shell explodes in midair and breaks up into many fragments. Which of the following statements are true regarding conditions immediately before and immediately after the explosion:

Statement I. The total momentum of the fragments is equal to the original momentum of the shell.
Statement II. The total kinetic energy of the fragments is equal to the original kinetic energy of the shell.
[A] Statement I only
[B] Statement II only
[C] Both Statement I and Statement II
[D] Neither statement is true.
3. A moving toy car of mass 0.5 kg moves to the right. Just before it collides with a stationary toy truck of mass 0.8 kg , the car's speed is $2 \mathrm{~m} / \mathrm{s}$. The two toy cars stick to each other.
a) Calculate the total momentum of the system before collision.
b) Calculate the speed of the two cars moving together immediately after collision
4. A moving toy car of mass 0.4 kg moves to the right. It collides with a stationary toy truck of mass 0.75 kg . After collision, the two toy cars stick to each other, and move together with a speed of 0.4 $\mathrm{m} / \mathrm{s}$ to the right.
a) What is the total momentum of the system (two cars) immediately after collision?
b) What was the speed of the moving toy car before collision?
5. A 2 g bullet travelling at $700 \mathrm{~m} / \mathrm{s}$ strikes and gets embedded in a 400 g block of wood. What is the speed of the wood + bullet immediately after the collision?
6. Abby and Dan are standing still in the middle of an ice rink. They push off against one another. As a result, Abby (mass $=45 \mathrm{~kg}$ ) acquires a speed of $0.4 \mathrm{~m} / \mathrm{s}$. What speed does Dan (mass $=60 \mathrm{~kg}$ ) acquire?
7. A 40 kg child running at a speed of $4 \mathrm{~m} / \mathrm{s}$ jumps into a 10 kg stationary wagon. Calculate the speed of the wagon + child immediately after the child jumps into the wagon.

8. A $60-\mathrm{kg}$ swimmer dives from a $200-\mathrm{kg}$ raft with a horizontal speed of $2 \mathrm{~m} / \mathrm{s}$. The raft is initially at rest. What is the speed of the raft immediately after the diver jumps?
9. A stationary bomb of mass 4 kg explodes into two fragments: one fragment of mass 1.5 kg flies to the right with a speed of $52 \mathrm{~m} / \mathrm{s}$. Calculate the speed of the other fragment.
10. A 500 kg cannon fires a 4 kg projectile with a velocity of $400 \mathrm{~m} / \mathrm{s}$ with respect to the ground. What is the recoil speed of the cannon?
11. A 15 kg shark moving at a speed of $8 \mathrm{~m} / \mathrm{s}$ swallows a 0.8 kg fish moving at a speed of $2 \mathrm{~m} / \mathrm{s}$ in the same direction as the shark. What is the speed of the shark immediately after swallowing the fish?
12. A 15 kg shark moving at a speed of $8 \mathrm{~m} / \mathrm{s}$ swallows a 0.8 kg fish moving at a speed of $2 \mathrm{~m} / \mathrm{s}$ toward the shark. What is the speed of the shark immediately after swallowing the fish?
13. A $50.0-\mathrm{kg}$ boy runs at a speed of $10.0 \mathrm{~m} / \mathrm{s}$ and jumps onto a cart as shown in the figure. The cart is initially at rest. If the speed of the cart with the boy on it is $2.50 \mathrm{~m} / \mathrm{s}$, what is the mass of the cart?

14. An $80-\mathrm{kg}$ astronaut carrying a $20-\mathrm{kg}$ tool kit is initially drifting toward a stationary space shuttle at a speed of $2 \mathrm{~m} / \mathrm{s}$. If she throws the tool kit toward the shuttle with a speed of $6 \mathrm{~m} / \mathrm{s}$ as seen from the shuttle, her final speed is
[A] $1 \mathrm{~m} / \mathrm{s}$ toward the shuttle.
[B] $1 \mathrm{~m} / \mathrm{s}$ away from the shuttle.
[C] $2 \mathrm{~m} / \mathrm{s}$ toward the shuttle.
[D] $4 \mathrm{~m} / \mathrm{s}$ toward the shuttle.
[E] $6 \mathrm{~m} / \mathrm{s}$ away from the shuttle.
15. A $58.5-\mathrm{kg}$ astronaut is floating toward the front of her stationary ship at $0.15 \mathrm{~m} / \mathrm{s}$. She wishes to stop moving. She decides to throw away the $2.50-\mathrm{kg}$ book she's carrying. What should the speed and direction of the book be to achieve her goal?
[A] $0.15 \mathrm{~m} / \mathrm{s}$, toward the front of the ship
[B] $0.30 \mathrm{~m} / \mathrm{s}$, toward the back of the ship
[C] $3.5 \mathrm{~m} / \mathrm{s}$, toward the back of the ship
[D] $1.8 \mathrm{~m} / \mathrm{s}$, toward the front of the ship
[E] $3.7 \mathrm{~m} / \mathrm{s}$, toward the front of the ship

## Practice 5.15: Impulse, Force and Momentum

1. Does an object moving at constant speed have momentum? Explain your answer.
2. Does an object moving at constant velocity have impulse? Explain your answer.
3. Which one of the following will require more impulse? (Note: The medicine ball is heavier than the soccer ball.)
[A] accelerating a soccer ball from rest to $10 \mathrm{~m} / \mathrm{s}$
[B] accelerating a medicine ball from rest to $10 \mathrm{~m} / \mathrm{s}$
[C] the impulse is the same for both.
Explain your answer.
4. Which one of the following is true concerning momentum?
[A] Momentum is a force.
[B] Momentum is a scalar quantity.
[C] The SI unit of momentum is $\mathrm{kg} \cdot \mathrm{m}^{2} / \mathrm{s}$.
[D] The momentum of an object is always positive.
[E] Momentum and impulse are measured in the same units.
5. A rock is dropped from a high tower and falls freely under the influence of gravity. Which one of the following statements is true concerning the rock as it falls?
[A] It will gain an equal amount of momentum during each second.
[B] It will gain an equal amount of kinetic energy during each second.
[C] It will gain an equal amount of speed for each meter through which it falls.
[D] It will gain an equal amount of momentum for each meter through which it falls.
[E] The amount of momentum it gains will be proportional to the amount of potential energy that it loses.
6. Which activity requires more impulse?
[A] landing from a jump while flexing the legs (bending at knee)
[B] landing from a jump while keeping the legs straight (locking knees)
[C] The impulse will be the same.
Explain your answer.
7. A stunt person jumps from the roof of a tall building, but no injury occurs because the person lands on a large, air-filled bag. Which one of the following best describes why no injury occurs?
[A] The bag provides the necessary force to stop the person.
[B] The bag reduces the impulse to the person.
[C] The bag increases the amount of time the force acts on the person and reduces the change in momentum.
[D] The bag decreases the amount of time during which the momentum is changing and reduces the average force on the person.
[E] The bag increases the amount of time during which the momentum is changing and reduces the average force on the person.
8. A ball A approaches and collides with a wall with a speed of $10 \mathrm{~m} / \mathrm{s}$. It rebounds with a speed of $5 \mathrm{~m} / \mathrm{s}$. A ball B approaches and collides with a wall with a speed of $30 \mathrm{~m} / \mathrm{s}$. It rebounds with a speed of $28 \mathrm{~m} / \mathrm{s}$. The two balls are identical. Answer the questions below:

| Ball A | a) Which ball has the greatest velocity change? |
| :--- | :--- | :--- |
| lall | b) Which ball has the greatest momentum change? |

9. In the graphs below you are shown how the velocities of two identical cars $A$ and $B$ change with time. Answer the questions below:

a) Which car has the greatest velocity change?
b) Which car has the greatest momentum change?
c) Which car has the greatest impulse?
d) Which car has the greatest force applied to it?

## Additional Problems:

10. Doug and Cory decide to go bungee jumping. They want to jump off a high bridge and live to talk about it. They agree they should tie one end of a cord of some sort around their waist/legs and attach the other end to the bridge. Doug says they should use a stretchy rubber (bungee) cord. Cory says they should use a strong metal cable because it will not break. Who is right and why? (picture fro life123.com)

11. When firefighters have to rescue persons from tall buildings on fire, they are forced sometimes to use safety nets. Explain why it is safe to use a safety net when jumping from a high point. (piture from nyworkerscompensationalliance.org)

12. What would be more damaging for your car: driving into a concrete wall, or driving at the same speed into a head on collision with another car traveling toward you at the same speed?
13. If you're about to catch a fast baseball with your bare hand, you extend your hand forward and then you let your hand move backward after you make contact with the ball. Explain your motion using physics.
14. Why would a ballet dancer prefer a wooden floor to a hard floor with little or no "give"?
15. What is the difference between the two water wheels shown below? Which one do you think is more efficient? Explain your answer. (picture from http://www.fordendwatermill.co.uk/guide.html)


Overshot


Undershot

Physics Crossword Puzzle
(from the Book of Phyz by Dean Baird)


## CLUES

## ACROSS

8 One billionth prefix
11 Giving human characteristics to inanimate objects
15 What the ugly duckling became

## 17 Calculator manufacturer, not Casio

19 Momentum is measured in kilogram-_?_s per second
20 When a single body is forcefully divided into two or more parts
23 Van Gogh died with very little money and only one _?
24 Equal to a $\mathrm{kg} \cdot \mathrm{m} / \mathrm{s}$

26 A military academy on the eastern seaboard
27 A type of collision in which kinetic energy is conserved
30 1,000,000,000,000 prefix
31 Unit of electrical resistance
$3379 \% \mathrm{~N}_{2}, 19 \% \mathrm{O}_{2}$
34 Impact force is inversely proportional to impact _?_
35 A word that implies division by time
37 F = ma is not the way Newton revealed his_?_ law in the Principia
39 Consume edible matter
41 Italian nuclear physicist Enrico; $10^{-15} \mathrm{~m}$
43 Deux en Español
46 An event in which two bodies undergo an abrupt interaction
49 Is credited with developing the principle of conservation of momentum
51 Greek letter; "British" unit of pressure
$55 \mathrm{HCl}, \mathrm{NaCl}, \mathrm{H}_{2} \mathrm{O}, \mathrm{CO}_{2}$, and $\mathrm{C}_{6} \mathrm{H}_{12} \mathrm{O}_{6}$ are chemical _? s
56 Momentum is conserved in any system in which no _? forces act
57 Chimpanzee, gorilla, orangutan, _?_
58 The result of unbalanced, external force
59 Eastern Indian stringed musical instrument that relies on resonance
60 One piece of the result of an explosion
62 The kinds of perfect conditions under which introductory physics occurs
63 The answer to \#34 across spelled backwards

## DOWN

1 Bay State; F =
2

3 A principle that applies to a quantity that does not change
4 Colorful _?_; blue ion element
5 Joule is the MKS unit of energy, _? is the cgs unit of energy
6 _?_gate timer
7 One one-thousandth of a second

9 Angle measuring smaller than $\pi / 2$ radians
10 Sopranos, tenors, orchestra, elaborate costumes and classic story lines; small window on a car
$12 \mathrm{~F}=\Delta \mathrm{p} / \Delta \mathrm{t}$ is _? ${ }_{-}$'s second law
$13 \mathrm{p}=$
14 The product of mass and velocity; inertia in motion
16 The rate of change in momentum
18 Half of the respiration process
21 Used to separate white light into colors
22 A sticky collision
25 Force/area in solids
28 Cyan, yellow, and magenta are primary _?_s in printing
29 Book of Phyz author
32 Momentum was regarded by Newton as "quantity of _?_"
$3610^{15}$ prefix
38 1980's Barry Levinson feature film about life in Baltimore in the early 1960's
$40(\mathrm{~N} \cdot \mathrm{~s}) /(\mathrm{m} / \mathrm{s})$
42 Developers and administrators of the SAT, SAT II, and AP Exams, among others
44 The tangent of $\pi / 4 \mathrm{rad}$
45 A hypothesis that has been verified through extensive testing over a long period of time
47 Change in momentum
48 If you increase the _? time, you decrease the _? force
49 The name of the Greek letter commonly used to denote change
50 1970's/80's UK band that enjoyed hits with "Tempted" and "Pulling Mussles (From the Shell)"
52 The chemical nature of the noble gases
532000 was a leap _? , but 1700, 1800, and 1900 were not
54 Molten constituent of the Earth's mantle
55 The modern version of this woodwind is commonly made of metal
60 The Sunshine State
61 Yellowish precious metal

## Applications of Newton's Laws: Review

## Free Fall

Free fall refers to objects falling under the influence of gravity.
Acceleration $\mathrm{a}=\mathrm{g}=-9.8 \mathrm{~m} / \mathrm{s}^{2}$, where the negative sign indicates that the acceleration is downward.
Parameters and equations that apply to free fall:
$v_{i}$ is the initial vertical velocity
$v_{f}$ is the final vertical velocity
$\Delta t$ is the time
$a_{y}$ is the vertical acceleration (usually $=g$ )

Equations of Motion:
Equation \# 1: $v_{f}=v_{i}+a \Delta t$
Equation \# 2: $\Delta \mathrm{y}=\mathrm{v}_{\mathrm{i}} \Delta \mathrm{t}+\frac{1}{2} \mathrm{a}(\Delta \mathrm{t})^{2}$

Equation \# 3: $v_{f}^{2}-v_{i}^{2}=2 a \Delta y$

## Projectile Motion

Projectile motion refers to motion that has horizontal and vertical components. The horizontal component is determined by the horizontal velocity, and has no acceleration, $a_{\mathrm{x}}=0$. The vertical motion is determined by gravity, just as in free fall.

Parameters and equations that apply to two-dimensional trajectories:

| Horizontal motion: | Vertical motion: |
| :--- | :--- |
| $v_{i x}$ is the initial horizontal velocity | $v_{i y}$ is the initial vertical velocity |
| $v_{f x}$ is the final horizontal velocity | $v_{f y}$ is the final vertical velocity |
| $a_{x}$ is the horizontal acceleration (usually $=0$ ) | $a_{y}$ is the vertical acceleration (usually $=g$ ) |
| $\Delta t$ is the time, which is the same for both hori- |  |
| zontal and for vertical motion |  |
|  | $v_{f y}=v_{i y}+a_{y} \Delta t$ <br> $v_{f x}=v_{i x}$ (since $\left.a_{x}=0\right)$ <br> $\Delta x=v_{i x} \Delta t$ |
|  | $\Delta y=v_{i y} \Delta t+\frac{1}{2} a_{y}(\Delta t)^{2}$ |
|  | Since $a_{y}=g=-9.8 m / s^{2}$ in SI units, |
|  | $v_{f y}=v_{i y}+(-9.8) \Delta t$ |
|  | $\Delta y=v_{i y} \Delta t+\frac{1}{2}(-9.8)(\Delta t)^{2}$ |

## Circular Motion

An object that moves with a constant speed in a circular path is said to have uniform circular motion. Uniform, because its speed does not change; circular because it travels in a circle of fixed radius ( $r$ ).

Parameters and Equations that apply to circular motion
$\mathrm{T}=$ period of motion (the time the object takes to complete one complete circle)
$f=$ frequency, the number of cycles per second
$v=$ speed, or magnitude of the tangential veloc-
ity
$r=$ radius of circle

## Momentum

Momentum is defined as mass x velocity. The total momentum is conserved in all collisions.

## Parameters and Equations that apply to momentum

$m_{1}$ and $m_{2}$ are the masses of the colliding ob- $\quad$ Momentum: $p=m v$
jects
$v_{l i}$ and $v_{2 i}$ are the initial velocities of $m_{1}$ and $m_{2}$
$v_{l f}$ and $v_{2 f}$ are the final velocities of $m_{1}$ and $m_{2}$
$\Delta t=$ time over which collision occurs
$F=$ average force transferred by collision

Speed of object undergoing uniform circular motion: $\mathrm{v}=2 \pi \mathrm{r} / \mathrm{T}$

Frequency: $f=1 / T$
Centripetal force: $\mathrm{F}=\mathrm{mv}^{2} / \mathrm{r}$

Change in momentum: $\Delta p$
Conservation of momentum: $m_{1} v_{1 i}+m_{2} v_{2 i}=$ $m_{1} v_{l f}+m_{2} v_{2 f}$
Impulse: $F \Delta t=\Delta p$


[^0]:    * she catches it at same height as she tosses it.

