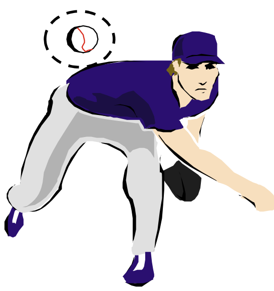
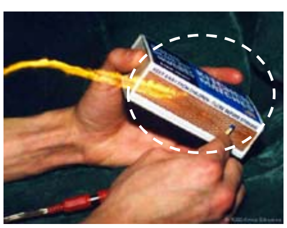
Reading Page: Work

Work is a common word in the English language, and it has many meanings. If one checks a dictionary, the following definitions can be found for work:

1. Physical or mental effort; labor.
2. The activity by which one makes a living.
3. A task or duty.
4. Something produced as a result of effort, such as a work of art.
5. Plural works: The essential or operating parts of a mechanism.
6. The transfer of energy to a body by application of a force.

The correct definition of work as used in physics is the last one: Work is the amount of energy transferred (through *working*) from the environment to a system, or from a system to the environment, by the application of forces to the system. Once the energy has been transferred to the system, it can appear in many forms. Exactly what form it takes depends on the details of the system and how the forces are applied. Below you are given a few examples of energy transfers due to work. We use *W* as the symbol for work. See examples below:

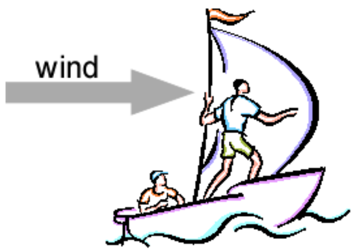
Pitching a baseball 	*The system*: The ball + earth. *The environment*: The athlete This is an open system. As the athlete pushes on the ball to get it moving, he is applying a force on the ball forcing it to move along the force, and thus doing work on the system. That is, he is transferring energy from himself to the ball. The energy transferred to the system appears as kinetic energy. The transfer: $W \rightarrow E_K$
Slingshot 	*The system*: The slingshot + earth. *The environment*: The boy. This is an open system. As the boy pulls back on the elastic band, he applies a force on the slingshot forcing it to move (stretch) along the direction of the force, and thus does work on the system, increasing its elastic potential energy. The transfer: $W \rightarrow E_{elastic}$
Lighting a match 	*The system*: The match + matchbox. *The environment*: The hand This is an open system. The hand does work on the system as it quickly pulls the match across the box. It forces the match to move in the direction of the applied force thus increasing its thermal energy due to friction. The match head becomes hot enough to ignite. The transfer: $W \rightarrow E_{thermal}$

Notice that in each of the above examples, the environment applies a force while the system moves in the direction of the force (undergoes a displacement). Energy is transferred as work *only* when the system moves along the direction of the force while the force acts. A force applied to a stationary object, such as when you push against a wall, transfers NO energy to the object and thus does NO work. It is also possible to convert work into gravitational potential, electric, or even chemical energy.
The key points to remember are:

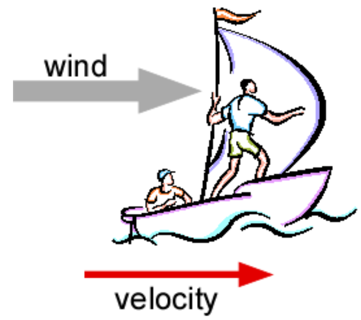
- work is the transfer of energy to or from a system by the application of external forces
- a system must undergo a displacement for this energy to be transferred.

Figuring out how much work is done

Let's examine an example of you in a boat.



A. Let's assume that there is no friction between the boat and the water. Initially the boat is not moving, and thus the system has no kinetic energy. Suddenly, the wind (a force from outside the system) starts blowing. In this case, work is done on the system and the energy of the system changes.



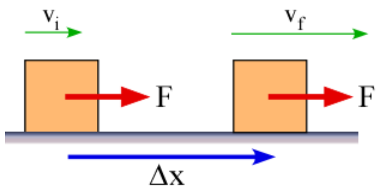
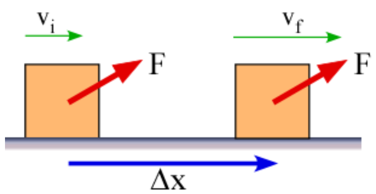
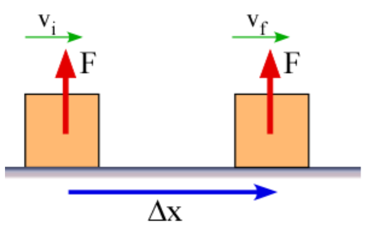
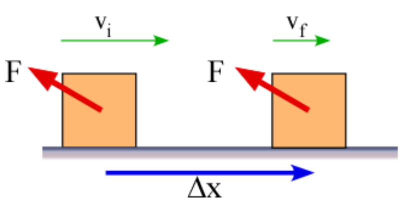
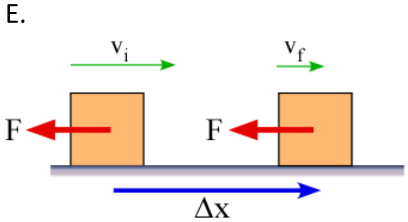
B. As the result of the wind blowing and pushing against the sails, the boat will begin to move and eventually speed up. The kinetic energy of the system increases. In terms of energy transfers, we would say that the energy of the system has increased because of the work done on the system by the force of the wind.

What determines how much work is done by the force of the wind? Let's assume that the force of the wind is constant. First, if the wind pushes with a stronger force, the sailboat speeds up more rapidly, and the change in its kinetic energy is greater than with a weaker force. Thus, *the stronger the force, the greater the work done.* Second, the greater the distance, over which the wind pushes the boat, the faster the sailboat goes, and the more its kinetic energy increases. This implies a greater transfer of energy. Therefore, *the bigger the displacement, the greater the work done.* This suggests that the amount of energy transferred into a system by a force —that is, the amount of work done by that force —depends on both the magnitude of the force, F, and the displacement Δx of the system under the action of this force. Your experiments have established that the amount of work done by is proportional to both F and Δx. For the simplest case described above, where the force is constant and points in the direction of the object's displacement, the expression for the work done is found to be

$$W = F \cdot \Delta x$$

The unit of work, that of force multiplied by distance, is N·m. This unit has its own name, the joule (J). Since work is simply energy being transferred, the joule is also the unit of all forms of energy. Note that work is a scalar quantity, it does not have a direction. However, since energy can be transferred both into and out of a system, positive and negative signs for work will be used to reflect this.

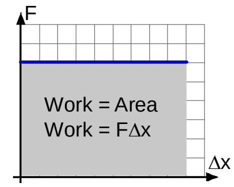
The table below shows you how to analyze work for different situations. The system in this case is a box that moves across a horizontal frictionless surface.

A. 	Maximum energy is transferred to the system through work because force is in the direction of motion. The kinetic energy of the object increases the most. $W > 0$
B. 	Only the component of force parallel to the direction of motion does work and thus contributes to transferring energy into the system. The object has a smaller increase in kinetic energy in this case than in case A. $W > 0$
C. 	There is no component of force F acting along the displacement of the object, therefore force F does no work. There is no energy transferred to the system (the box does not change its velocity). $W = 0$
D. 	The component of force parallel to the direction of motion is in the opposite direction to the direction of motion of the object. The object slows down and its kinetic energy decreases thus work done by force F is negative (energy is transferred out of the system). $W < 0$
E. 	Force F is in the opposite direction to the direction of motion. Maximum energy is transferred out of the system through work done by force F. Energy is decreasing, thus work is negative. $W < 0$

Forces That Do No Work

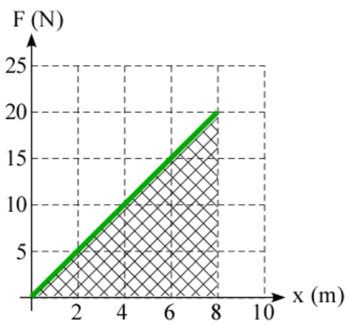
If the object undergoes no displacement while the force acts, no work is done. For example, the force with which you push against a wall does no work because the wall is not moving. You may get tired if you push hard and for a long time but from a physical point of view, the force you apply does no mechanical work.

Calculating Work as area under a Force vs Displacement graph



When you have a constant force acting on an object, the object moves under the action of this force and the work done by the force can be calculated as the area under the Force vs displacement graph, as seen in the lab. Or, we can calculate work as $W = F \cdot \Delta x$ (which represents the area under the graph).

How can we calculate work when the force applied to an object is not constant? The expression $W = F \cdot \Delta x$ is valid only when the applied force is constant but we learned that we can calculate work as the area under the graph. For example, let's consider a box moving across a floor. The applied force changes as shown in the graph below. Work done by this variable force can be calculated as:



$$W = \text{area of triangle} = \frac{(\text{base})(\text{height})}{2}$$

Since the force and displacement are both represented by standard units, we can easily calculate work as:

$$W = \frac{(8\text{ m})(20\text{ N})}{2} \Rightarrow W = 80\ \underbrace{\text{N}\cdot\text{m}}_{=\text{J}} = 80\text{ J}$$

Calculating Net/Total Work

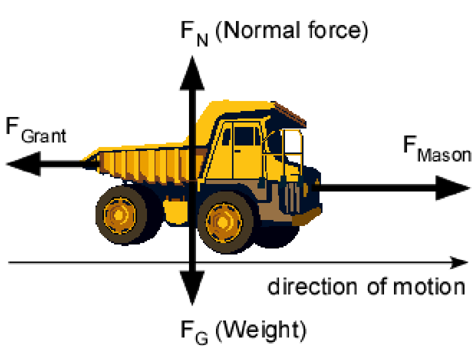
If more than one external force acts on the system of study, and the system moves, then one can calculate the net work is the net force acting on the object times the displacement of the object along the net force. An alternate method to calculate the net work is to calculate the work done by *each* force acting on the system and *sum up* all these works.

Example 1:

Grant and Mason are playing with a toy truck of mass m = 2 kg. Mason pulls on the truck to the right with a force of 25 N, and Grant pulls on the truck to the left with a force of 15N. As a result, the truck moves a distance of 2 m to the right. Calculate the net work done on the truck. Neglect friction with the ground.

Solution:

System: truck.
This is a non-isolated system and every force acting on it is an external force.

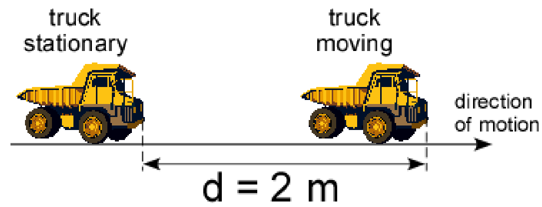


Let us calculate the work done by each individual force and sum them up. First, identify all the external forces acting on the truck. The normal force and the weight (gravitational force) are perpendicular to the direction of motion of the truck and therefore the work done by these forces is zero (there is no displacement of the truck under the action of these forces).

$W_{F_G} = 0$ and $W_{F_N} = 0$

Now when calculating work done by force applied by Mason and Grant, one must realize that F_{Mason} is applied in the same direction as the direction of motion of the truck, but F_{Grant} is applied in the opposite direction. If we consider the positive direction the same as the direction of motion of the truck, then F_{Mason} is positive and F_{Grant} is negative.

$$W_{F_{Mason}} = (F_{Mason})(\Delta x) \Rightarrow W_{F_{Mason}} = (25\text{ N})(2\text{ m}) = 50\text{ J}$$
$$W_{F_{Grant}} = (F_{Grant})(\Delta x) \Rightarrow W_{F_{Grant}} = (-15\text{ N})(2\text{ m}) = -30\text{ J}$$



The net work done by all forces acting on the truck will be:

$$W_{net} = W_{F_G} + W_{F_N} + W_{F_{Mason}} + W_{F_{Grant}}$$
$$W_{net} = (0) + (0) + (50\text{ J}) + (-30\text{ J})$$
$$W_{net} = 20\text{ J}$$

What happens with all the work? When Mason and Grant start pulling on the car, the car is stationary: the kinetic energy of the car is zero. The gravitational potential energy of the car (being on the ground) is also zero and it is not changing. But the kinetic energy of the car increases because it starts moving. Thus, energy was transferred into the system through working and the amount of energy that was transferred equals the calculated net work.

Reading Page: Gravitational Potential Energy

When two or more objects in a system interact, it is sometimes possible to store energy in that system in a way that the energy can be easily recovered. For example, the earth and a ball interact by the gravitational force between them. If the ball is lifted up into the air, energy is stored in the ball-earth system, energy that can later be recovered as kinetic energy when the ball is released and falls. Similarly, a bow and arrow form a system in which the two interact. The arrow pushes against the bow and deforms it, thus storing energy in the system that later can be recovered. When the arrow is released, the stored energy is transformed into kinetic energy of motion for the arrow as the deformation of the bow disappears. This sort of stored energy is called potential energy, since it has the potential to be converted into other forms of energy such as kinetic or thermal energy.

The forces due to gravity and springs are special in that they allow for the storage of energy. Other interaction forces do not. When a crate is pushed across the floor, the crate and the floor interact via the force of friction, and the work done on the system is converted into thermal energy. But this energy is not stored up for later recovery—it slowly diffuses into the environment and cannot be recovered.

Gravitational Potential Energy

Gravitational potential energy is stored energy associated with an object's height above the ground. For example, as a roller coaster ascends the track, energy is stored as increased gravitational potential energy with respect to the ground. This energy is transferred into the system through work done to lift the rollercoaster. As the roller coaster descends, this stored energy is converted into kinetic energy.

There is a direct relation between gravitational potential energy and the mass of an object; the more mass an object has, the greater its gravitational potential energy. This is because more work must be done to lift an object with more mass. The work is done against the force of gravity acting on the object in order to change the position of the object with respect to the ground. There is also a direct relation between gravitational potential energy and the vertical height of an object; the higher an object is lifted, the more gravitational potential energy it has. Once again, more work must be done to lift an object on a larger distance. Therefore we can calculate the gravitational potential energy of a system with respect to the ground (the zero of the gravitational potential energy) as:

$$E_g = mgy$$

where *m* is the mass of the object, *g* is the gravitational acceleration, or the gravitational field strength (with a value of 9.8 m/s^2), and *y* is the vertical position with respect to an arbitrary zero level. The zero is chosen to be convenient for the solution of the problem. For instance, the zero level might be assigned as the base of a cliff for a projectile being thrown from the top of the cliff or the zero level might be the floor for a marble rolling off a table onto the ground.

$E_g = mgy$ where *m* is measured in kg, *g* equals 9.8 m/sec^2, and *y* is in meters.

Units: $[\text{kg}]\left[\frac{\text{m}}{\text{s}^2}\right][\text{m}]=[\text{kg}]\left[\frac{\text{m}^2}{\text{s}^2}\right]=[\text{N}][\text{m}]=$ J is a Joule (J)

Or, from

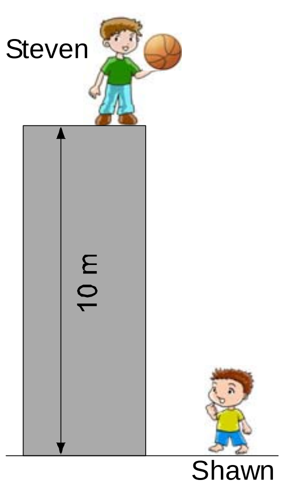
$$E_g = \underbrace{(mg)}_{weight} y \Rightarrow [\text{J}]=[\text{N}][\text{m}]$$

The expression *mg* represents the object's weight or the force of gravity acting on it. Forces are measured in Newtons. Thus $[\text{N}][\text{m}]$ also equals a Joule (J).

To determine the gravitational potential energy of an object, a zero height position must first be arbitrarily assigned. Typically, the ground is considered to be a position of zero height, but this is simply an arbitrarily assigned position which most people agree upon. Since many of the labs are done on tabletops, the tabletop is a convenient zero height position. The potential energy of an object is then based upon its height relative to the tabletop. For example, a coffee cup sitting on top of the table has a potential energy which can be measured based on its height above the table top. This gravitational potential energy will be positive because the position of the coffee cup is positive. A ball found under the table will have a negative gravitational potential energy with respect to the table because its position is negative. Since the gravitational potential energy of an object is directly proportional to its height above or below the zero position, a doubling of the height will result in a doubling of the gravitational potential energy. A tripling of the height will result in a tripling of the gravitational potential energy.

Notice that the potential energy of an object is a scalar quantity whose value is directly proportional to the object's mass and is also directly proportional to the object's height above (or below) an arbitrarily chosen zero level. The gravitational potential energy can have positive or negative values, depending on the reference frame chosen.

Example 1



Steven and Shawn are playing with a 0.2 kg ball. Steven is on top of a building that is 10 meters tall. He has the ball in his hand and is ready to let it drop. Shawn, waiting at the base of the building, is ready to catch the ball. In all calculations take g = 9.8 m/s^2.

a) What is the gravitational potential energy of the ball with respect to Steven before being released?

b) What is the gravitational potential energy of the ball with respect to Shawn before being released?

c) What is the gravitational potential energy of the ball with respect to Steven just before it hits the ground?

d) What is the gravitational potential energy of the ball with respect to Shawn just before it hits the ground?

Solution:

System: ball + Earth. This is an isolated system. *Initial position*: ball at the top of the building (where Steven holds it) *Final position*: ball on the ground (where Shawn catches it)	Given quantities: $m = 0.2$ kg $y_i = 10$ m $y_f = 0$ m $g = 9.8$ m/s^2

Zero of gravitational potential energy: top of building (where Steven is).	*Zero of gravitational potential energy*: ground (where Shawn is).
a) Before being released, the position of the ball with respect to Steven is 0 m. $E_g = mgy$ $E_g = (0.2)(9.8)(0) \Rightarrow E_g = 0$ J	b) Before being released, the position of the ball with respect to Shawn is 10 m. $E_g = mgy$ $E_g = (0.2)(9.8)(10) \Rightarrow E_g = 19.6$ J
c) When the ball hits the ground, its position with respect to Steven is (- 10 m). $E_g = mgy$ $E_g = (0.2)(9.8)(-10) \Rightarrow E_g = -19.6$ J	c) When the ball hits the ground, its position with respect to Shawn is 0 m. $E_g = mgy$ $E_g = (0.2)(9.8)(0) \Rightarrow E_g = 0$ J

Note: in all cases, the units must match. Always make sure that the physical quantities you are using are expressed in standard units: mass in kg, gravitational acceleration in m/s^2 and position in m.

Observation: depending on where one chooses the zero of the gravitational potential energy, one can get zero, a positive or a negative value for the gravitational potential energy.

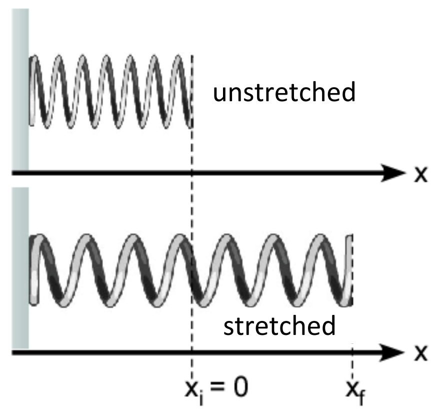
Reading Page: Elastic Potential Energy

Elastic Potential Energy

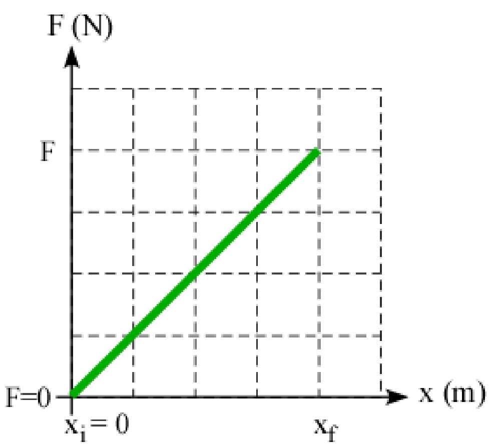
Energy can be stored in a compressed or extended spring as elastic (or spring) potential energy. We can figure out how much energy is stored in a spring by using an external force to slowly compress or stretch the spring. This external force does work on the spring, transferring energy to the spring. Since only the elastic potential energy of the spring is changing, it means that all the work done by the external force is used to transfer energy into the system. We can find out how much elastic potential energy is stored in the spring by calculating the amount of work needed to compress the spring. The conservation of energy law can be written as:

$W = \Delta E_e$

Work done by any force can be calculated as the area under the force vs displacement graph. Remember that the force required to stretch a spring is equal to the elastic force which is directly proportional with the displacement of one end of the spring, $F = kx$ where k = constant of elasticity (related to the type of spring one uses) and x is measured from zero.



The graph below shows the force vs displacement for a spring.



The slope of the graph represents the elastic constant (k) of the spring. The work done by this force can be calculated as the area under the graph. Remember that this area is a triangle. Therefore:

$$W = \text{Area} = \frac{(\text{base})(\text{height})}{2}$$

$$W = \frac{(x_f - x_i)(F_e)}{2}$$

Substitute

$$F_e = k(x_f - x_i)$$

$$\Rightarrow W = \frac{(x_f - x_i)k(x_f - x_i)}{2} = \frac{k(x_f - x_i)^2}{2}$$

$$\Rightarrow W = \frac{k\left(\Delta x\right)^2}{2}$$

Therefore the elastic energy stored in the spring will be: $\Delta E_e = \frac{k\left(\Delta x\right)^2}{2}$

From this expression one can see that the elastic potential energy is always positive because the elastic constant is always a positive number and the displacement being squared also gives you a positive value. This implies that stretching a spring by Δx or compressing the spring by the same amount ($-\Delta x$) stores the same amount of energy.

Example 1

Students in Ms. Zinszer's class are trying to identify the weakest and the strongest springs in their classrooms. Using spring scales, they collected the following data for the three springs they had in their classroom:

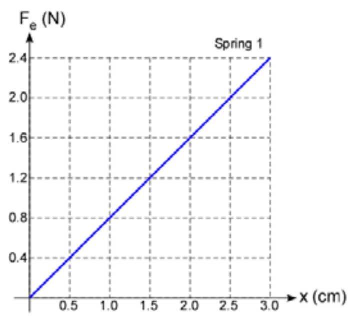
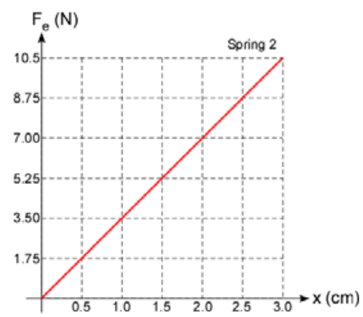
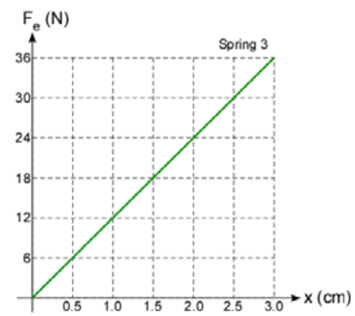
Spring extension Δx (cm)	Spring 1 Force (N)	Spring 2 Force (N)	Spring 3 Force (N)
0.5	0.4	1.75	6
1.0	0.8	3.5	12
1.5	1.2	5.25	18
2.0	1.6	7.0	24
2.5	2.0	8.75	30
3.0	2.4	10.5	36
3.5	2.8	12.25	42

a) Just by looking at the data, which spring is the strongest and which one is the weakest?
b) Calculate the constant of elasticity for each spring by plotting the data and finding the slope of that graph.
c) How much energy is stored in each spring when they are stretched 2.2 cm?
d) How much energy is stored in spring 1 when it is stretched from 1 cm to 2 cm?

Solution:

a) From the data one can see that spring 3 has the highest values for the forces required to stretch it and therefore it is the strongest spring. Spring 1 has the smallest values for force and therefore it is the weakest spring.

b) Let us plot the Force F_e (N) vs deformation Δx (cm) for each spring and find the slope of each graph. Remember that slope is calculated as rise/run. Note: even though the three graphs look the same, the scale for the forces is different!

<table>
<tr><td>Spring 1</td><td>Spring 2</td><td>Spring 3</td></tr>
<tr><td>
</td><td>
</td><td>
</td></tr>
<tr><td>$k_1 = \frac{rise}{run} \Rightarrow k_1 = \frac{2.4 \text{ N}}{3 \text{ cm}} = \frac{2.4 \text{ N}}{0.03 \text{ m}}$
$k_1 = 80 \text{ N/m}$</td><td>$k_2 = \frac{rise}{run} \Rightarrow k_2 = \frac{10.5 \text{ N}}{3 \text{ cm}} = \frac{10.5 \text{ N}}{0.03 \text{ m}}$
$k_2 = 350 \text{ N/m}$</td><td>$k_3 = \frac{rise}{run} \Rightarrow k_3 = \frac{36 \text{ N}}{3 \text{ cm}} = \frac{36 \text{ N}}{0.03 \text{ m}}$
$k_3 = 1200 \text{ N/m}$</td></tr>
</table>

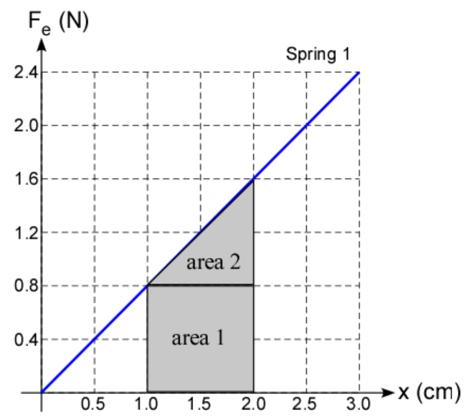
c)

When stretched 2.2 cm, the elastic potential energy stored in (don't forget to transform the cm in m):

spring 1 is:	spring 2 is:	spring 3 is:
$E_e = \frac{k_1(\Delta x)^2}{2}$ $E_e = \frac{(80 \text{ N/m})(0.022 \text{ m})^2}{2}$ $E_e = 0.01936 \text{ Nm}$ $E_e = 0.01936 \text{ J}$	$E_e = \frac{k_2(\Delta x)^2}{2}$ $E_e = \frac{(350 \text{ N/m})(0.022 \text{ m})^2}{2}$ $E_e = 0.0847 \text{ Nm}$ $E_e = 0.0847 \text{ J}$	$E_e = \frac{k_3(\Delta x)^2}{2}$ $E_e = \frac{(1200 \text{ N/m})(0.022 \text{ m})^2}{2}$ $E_e = 0.2904 \text{ Nm}$ $E_e = 0.2904 \text{ J}$

d) One can calculate the energy stored in a spring as the area under the force vs stretch. Therefore we can easily calculate the energy stored when the spring is stretched from 1 cm to 2 cm as the area under the F vs x curve: a rectangle + a triangle (as shown in the figure below).

$$E_e = \text{area 1} + \text{area 2}$$
$$E_e = (\text{base})(\text{height}) + \frac{1}{2}(\text{base})(\text{height})$$
$$E_e = (1 \text{ cm} \times 0.8 \text{ N}) + \frac{1}{2}(1 \text{ cm} \times 0.8 \text{ N})$$
$$E_e = (0.01 \text{ m} \times 0.8 \text{ N}) + \frac{1}{2}(0.01 \text{ m} \times 0.8 \text{ N})$$
$$E_e = 0.012 \text{ Nm}$$
$$E_e = 0.012 \text{ J}$$



Note: the energy stored in the spring when it is stretched from 0 to 1 cm is equivalent to area 2 on the graph above. To stretch the spring from 1 to 2 cm requires 3 times as much energy: area 2 + area 1 which is twice area 2.

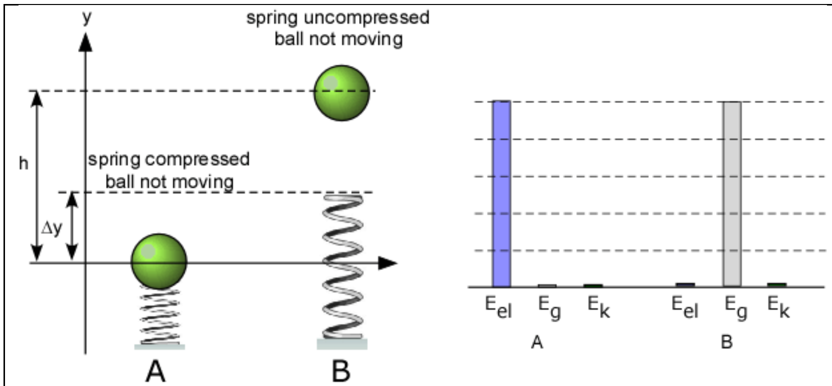
Example 2

A 0.2 kg ball is placed on a vertical spring with a spring constant of 200 N/m. The spring is compressed 0.5 m and then released. What is the maximum height that the ball will reach?

Solution:

System: ball + spring + Earth. This is an isolated system. *Initial position*: spring compressed, ball stationary *Final position*: spring not compressed, ball not moving (at highest point) *Zero of gravitational potential energy*: top of the spring in compressed position.	Given quantities: $m = 0.2 \text{ kg}$ $k = 200 \text{ N/m}$ $\Delta y = 0.5 \text{ m}$ $g = 9.8 \text{ m/s}^2$

Start the problem by making a drawing. Choose the zero for the gravitational potential energy to be at the top of the spring. Therefore in the initial state (A), the system (spring + ball + Earth) has only elastic potential energy. In the final state (B), when the spring is not compressed anymore, and the ball reached the maximum height h (where it stops and reverses its motion), the system has only gravitational potential energy.

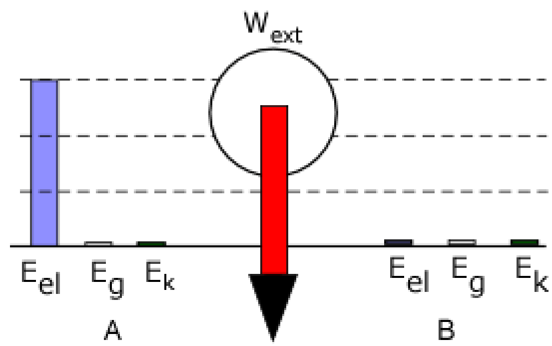


$$E_{totalA} = E_{totalB}$$
$$E_{eA} + \underbrace{E_{gA}}_{=0} + \underbrace{E_{kA}}_{=0} = \underbrace{E_{eB}}_{=0} + E_{gB} + \underbrace{E_{kB}}_{=0}$$
$$\Rightarrow \frac{1}{2}k(\Delta y)^2 = mgh$$
$$\Rightarrow \frac{1}{2}(200)(0.5)^2 = (0.2)(9.8)(h)$$
$$\Rightarrow 25 = 1.96h$$
$$\Rightarrow h = 12.75 \text{ m}$$

The system is isolated, there are no external forces acting on it, and therefore there will be no energy transfer through working or heating into or out of the system. Therefore, all the elastic potential energy was transformed into gravitational potential energy which we can calculate from the conservation of energy law.

Alternate solution:

Now let us assume that we choose as our system only the ball and the spring, and earth is not part of the system anymore: there is no gravitational potential energy associated with this system. In this case, the gravitational force acting on the ball is an external force that transfers energy into the system through working. The energy bar diagram will look like below:



The work done by the gravitational force is negative because the gravitational force is oriented (down) in the opposite direction to the motion of the ball (up). Also, energy is transferred out of the system, and not into the system.

$$E_{totalA} = E_{totalB}$$
$$E_{eA} + \underbrace{E_{gA}}_{=0} + \underbrace{E_{kA}}_{=0} - W_{ext} = \underbrace{E_{eB}}_{=0} + \underbrace{E_{gB}}_{=0} + \underbrace{E_{kB}}_{=0}$$
$$\Rightarrow \frac{1}{2}k(\Delta y)^2 - \left(\underbrace{mg}_{F}\, h\right) = 0$$
$$\Rightarrow \frac{1}{2}(200)(0.5)^2 - (0.2)(9.8)(h) = 0$$
$$\Rightarrow 25 = 1.96h$$
$$\Rightarrow h = 12.75 \text{ m}$$

The result obtained is the same because it does not depend on how one chooses the system or the origin of the reference frame.

Reading Page: Kinetic Energy

Kinetic energy is the energy of motion. An object which has motion, regardless of its direction, has kinetic energy. There are many forms of kinetic energy:

- translational (the energy due to motion from one location to another),
- rotational (the energy due to rotational motion),
- vibrational (the energy due to vibrational motion).

From here on, the phrase kinetic energy will refer to translational kinetic energy, the only type of kinetic energy that we will study. How can we calculate how much kinetic energy a moving object has? What does the kinetic energy depend on?

You are playing with two weights, a 1 lb one and a 10 lb one. You drop them both from the same height and they fall and hit the toes of your foot. Which one hurts more? The 10 lb one hurts more than the 1 lb one. Both weights started falling from the same height and therefore, when reaching your foot, both had the same speed (you have learned this previously). How come that the heavier weight hurts more? The kinetic energy of the weight when reaching your foot is transferred to your foot through working and therefore the more work means the more kinetic energy. That means that the heavier weight had more kinetic energy; thus one can conclude that kinetic energy of an object depends on its mass.

Now imagine that you play with two identical weights, 5 lb each. You drop one on your foot and from the same height you throw the other one down on your foot. Which one hurts more? The one you threw, because it reached your foot with a higher speed, and therefore a bigger kinetic energy. Thus one can conclude that the kinetic energy of an object also depends on its speed.

In conclusion, we can say that the amount of kinetic energy which an object has depends upon two variables: the mass (m) of the object and the speed (v) of the object. The following equation is used to represent the kinetic energy (E_k) of an object.

$$E_k = \frac{1}{2}mv^2$$

This equation shows that the kinetic energy of an object is directly proportional to the square of its speed. That means that if the speed of the object doubles, the kinetic energy will increase by a factor of four; for a threefold increase in speed, the kinetic energy will increase by a factor of nine; and for a fourfold increase in speed, the kinetic energy will increase by a factor of sixteen.

Kinetic energy is a scalar quantity; it does not have a direction. Like work and potential energy, the standard metric unit of measurement for kinetic energy is the Joule.

Example 1

a) Calculate the kinetic energy of a pumpkin of mass m = 1.4 kg when moving with velocity v = 2 m/s.

Solution:

$$E_k = \frac{1}{2}mv^2 \Rightarrow E_k = \frac{1}{2}(1.4)(2)^2$$

$$E_k = 2.8 \text{ J}$$

b) How much is the kinetic energy of the pumpkin when its velocity doubles?

Solution:

$E_k = \frac{1}{2}mv^2 = \frac{1}{2}(1.4)(4)^2$ The kinetic energy quadruples.

$E_k = 11.2 \text{ J}$

Example 2

Calculate the speed of an apple of mass m=0.5 kg that has a kinetic energy of 4 J.

Solution:

$$E_k = \frac{1}{2}mv^2$$
$$4 = \frac{1}{2}(0.5)(v)^2$$
$$4 = 0.25(v)^2$$
$$(v)^2 = \frac{4}{0.25} = 16$$
$$v = 4 \text{ m/s}$$

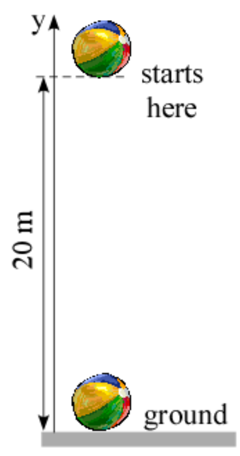
Units check:

$$E_k = \frac{1}{2}mv^2$$
$$[\text{E}]=[\text{m}][\text{v}^2]$$
$$[\text{J}]=[\text{kg}][\text{m}^2/\text{s}^2]$$

Example 3

A ball falls from 20 m above the ground. What is the speed of the ball just as it touches the ground?

Solution: System: ball + Earth (ground) Initial position: ball at position 20 m above the ground Final position: ball on the ground Zero of E_g: ground	Just as the ball starts falling, all the energy stored in the ball + earth system is gravitational potential energy. As the ball falls toward the ground, the gravitational potential energy transforms into kinetic energy (the position of the ball relative to the ground changes, it decreases, and its velocity increases). By the ball touches the ground all the gravitational potential energy of the ball has been transformed into kinetic energy. But the total energy of the system (ball + earth) is conserved because the system is isolated, there are no external forces acting on it.



$$E_{\text{total at top}} = E_{\text{total at bottom}}$$
$$E_g = E_k$$
$$mgy = \frac{1}{2}mv^2$$
Divide by m
$$gy = \frac{1}{2}v^2$$
$$(9.8)(20) = \frac{1}{2}(v)^2$$
$$392 = (v)^2$$
$$v = 19.8 \text{ m/s}$$

Units Check

$$mgy = \frac{1}{2}mv^2$$
$$[\text{kg}]\left[\frac{\text{m}}{\text{s}^2}\right][\text{m}]=[\text{kg}]\left[\frac{\text{m}^2}{\text{v}^2}\right]$$

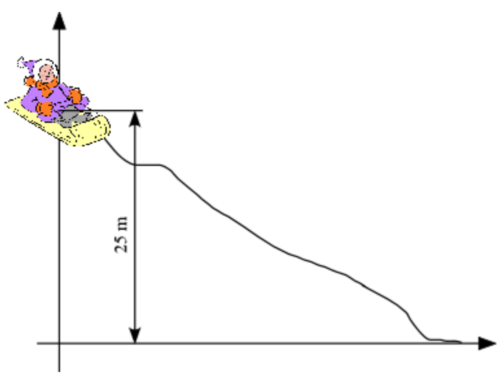
Example 4

A sled with a child in it, of combined mass 40 kg, is sliding down a long hill. At the top of the 25 m high hill the speed of the sled is 4 m/s. Ignore the friction between the sled and ground.
a) Calculate the total energy of the sled at the top of the hill.
b) Calculate the speed of the sled at the bottom of the hill.
c) Calculate the work done by the net force on the sled.

Solution:

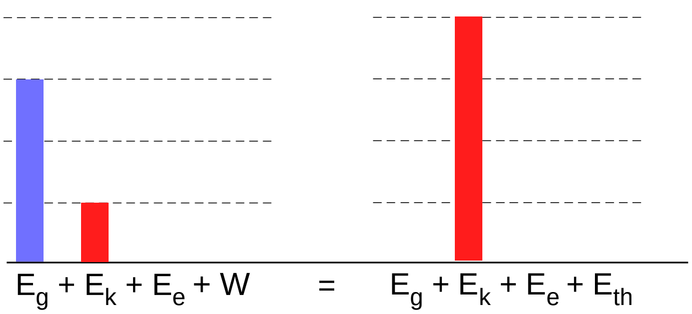
System: child + sled + earth. This is an isolated system, with no external forces acting on it, therefore the total energy of the system is conserved.
Initial position: sled at the top of the hill
Final position: sled at the bottom of the hill
Zero of E_g: bottom of the hill

a) When the sled is at the top of the hill, the total energy of the system (sled + child + earth) is made up of gravitational potential energy and kinetic energy.



$$E_{top} = E_g + E_k$$
$$E_{top} = mgy + \frac{1}{2}mv^2$$
$$E_{top} = (40)(9.8)(25) + \frac{1}{2}(40)(4)^2$$
$$E_{top} = 9800 + 320$$
$$E_t = 10120 \text{ J} = 10.12 \text{ kJ}$$

b) As the sled moves down the hill, its gravitational potential energy decreases and its kinetic energy increases (it is moving faster). Thus the gravitational potential energy transforms into kinetic energy. The system is isolated so no external forces act on the sled and therefore no energy transfers into or out of the system through working or heating, as shown in the energy bar graphs below.



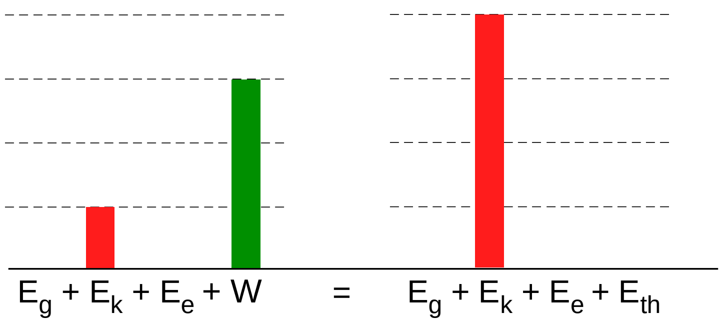
$$E_{top} = E_{bottom}$$
$$E_{top} = \underbrace{E_{g(bottom)}}_{=0} + E_{k(bottom)} = \frac{1}{2}mv_{bottom}^2$$
$$10120 = \frac{1}{2}(40)v_{bottom}^2 \Rightarrow (v_{bottom})^2 = 506$$
$$v = 22.5 \text{ m/s}$$

c) In order to calculate the net work done by the net force on the sled + child, we must define our system in a different way.

System: child + sled. This is not an isolated system. There are external forces acting on it. Therefore when calculating the total energy of the system, energy transfers into or out of the system (through working or heating) must be accounted for.
Initial position: sled at the top of the hill

Final position: sled at the bottom of the hill
Zero of E_g: ground

Because the earth is not part of our system, the sled has no gravitational potential energy. There is only kinetic energy at the top and the bottom of the hill. Because the kinetic energy at the bottom is larger (sled moving faster) than at the top, it is clear that there must be energy transferred into the system through working done by the net force. Therefore we must add the work done by the net force to the total energy in the initial state (top of the hill), as shown in the energy bar graphs below. Since there is no friction, there will be no thermal energy.



$$E_{k\ \mathrm{top}} + W_{net} = E_{k\ \mathrm{bottom}}$$

$$\frac{1}{2}mv_{top}^2 + W_{net} = \frac{1}{2}mv_{bottom}^2$$

$$\frac{1}{2}(40)(4)^2 + W_{net} = \frac{1}{2}(40)(22.5)^2$$

$$320 + W_{net} = 10125$$

$$W_{net} = 9805\ \mathrm{J}$$

This work is done by the gravitational force acting on the sled, the only external force that has a component along the direction of motion and it is the same as the gravitational potential energy.

Reading Page: Power

When we discussed work, we never mentioned how long it takes to do the work. The definition of work also does not tell us anything about the time required to do the work. From physics' point of view, the same amount of work is done when carrying a load up a flight of stairs, whether we walk up or run up. Also remember that work represents the change in energy but nowhere have we discussed how fast the energy of a system can increase or decrease through working. So why are we more tired when running up 3 flights of stairs in a few seconds than when walking up the same number of stairs in a few minutes, since the amount of work done is the same in both cases? If one considers that we moved with constant speed, then the change in the total energy of the system is the same in both cases but, in one case energy was changed faster and in the other case slower. To understand this difference, we need to talk about a measure of how *fast we can* store energy in a system or transfer the energy out of the system (through working). The physical quantity that allows one to measure this is called power.

Power is equal to the amount of work done over the time it takes to do it or more general the change in the energy of a system over that time it takes to change it:

$$P = \frac{\Delta E}{\Delta t} \text{ or } P = \frac{W}{\Delta t}$$

An automobile engine that delivers twice the power of another does not necessarily produce twice as much work or make cars go twice as fast as the less powerful engine does. Twice the power means it can do the same amount of work in half the time or twice the work in the same time. A more powerful engine can get an automobile up to a given speed in less time than a less powerful engine can.

The unit of power is the joule per second (J/s), also known as the watt (in honor of James Watt, the eighteenth-century developer of the steam engine). One watt (W) of power is expended when 1 joule of work is done in 1 second. One kilowatt (kW) equals 1000 watts. One megawatt (MW) equals 1 million watts. (One horsepower is the same as three-fourths of a kilowatt, so an engine rated at 134 horsepower is a 100-kW engine.)

Example 1

A forklift is used for placing heavy boxes on high shelves. A 25 kg box is lifted to a height of 6 m.
a) How much work did the forklift do in lifting the box with constant speed to that height?
b) If it takes 6 seconds to lift the box, how much power was used by the forklift? Express your answer in J/s, kW and horsepower.

Solution:

a) The work done by the forklift is against the force of gravity. Because the box is lifted at constant speed, the force applied by the forklift is the same as the force of gravity acting on the box. $\left.\begin{matrix} W = F \cdot h \\ F = mg \end{matrix}\right\} \Rightarrow W = mgh$ $W = (25)(9.8)(6)$ $W = 1470 \text{ J}$	b) The power used by the forklift is: $P = \frac{W}{\Delta t}$ $P = \frac{1470 \text{ J}}{6 \text{ s}} = 245 \text{ J/s}$ $P = 245 \text{ J/s} = 245 \text{ W} = \frac{245}{1000} \text{ kW} = 0.245 \text{ kW}$ $P = 0.245 \text{ kW} = \left(\frac{3}{4}\right)(0.245) \text{ hp} = 0.18375 \text{ hp}$

Table conversion for Power units:

1 horsepower (hp)	0.746 kW
1 horsepower (hp)	550 ft·lb/s
1 Watt (W)	1 J/s
1 Watt (W)	0.738 ft·lb/s
1 Btu/h	0.293 W

Online website for unit conversion:

http://www.onlineconversion.com/

http://www.unitconversion.org/

http://www.unit-conversion.info/

Reading Page: Energy

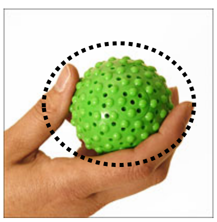
One of the greatest and most significant discoveries of science is that there is such a "thing" called energy. You have heard of some of the many forms of energy: solar energy, wind energy, or nuclear energy, but others may be new to you. All forms of energy are measured in Joules (J) (a Joule is N·m or $kg \cdot m^2/s^2$).

But what is energy? Energy is a conserved physical quantity that has the ability to produce changes in physical systems. In everyday language the word "conserve" means "to save" or "to use less of". In science, it has a different meaning. Energy is conserved means that energy cannot be created or destroyed; it can only be converted from one form to another, or transferred from one system to another. Energy conservation is one of the most fundamental and important laws of nature.

As we discuss energy, we must keep track of what happens to it. Energy can be transformed within a system, transferred between systems, or stored in a physical system.

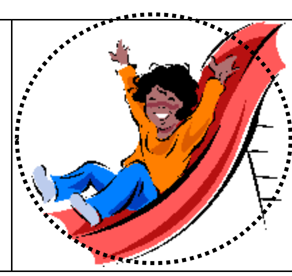
- **Energy Transformation**: a process where energy changes types within one system.
- **Energy Transfer**: a process where energy travels into or out of a system.
- **Energy Storage**: the "type" of energy that stays/it is stored in a system.

To analyze the energy of a system, one must first define a system. The object of interest for us is called *system* and everything outside the system is called the *environment*. Making an appropriate choice of system when analyzing energy can simplify things. A *process* is the change in the system, from an initial time (*initial state*) to some final time (*final state*).



Example: a ball that you hold in your hands and then let drop toward the ground may be defined as the *system* (surrounded by a dotted line). Everything except the ball is the *environment* (for example, the hand). *Initial state* for the ball may be chosen as the moment when you let it drop from your hands (at a certain height with respect to the ground). The *final state* for the ball may be chosen as the moment when it hits the ground.

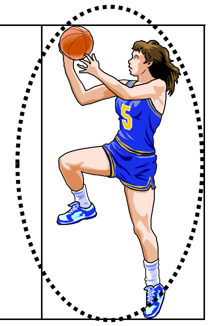
There can be interactions between objects in a system or between objects in the system and objects in the environment. Forces that objects inside the system exert on each other are called *internal forces*; forces that objects outside the system exert on the objects inside the system are called *external forces* (see examples below).

System A: girl plus slide *Internal forces*: - the force between the girl and the slide (normal) - the friction force between the girl and the slide. *External forces*: - gravitational force acting on the girl and the slide plus the normal force from the ground acting on the slide	

System B: girl plus basketball

Internal forces:
- the force between the girl and the basketball

External forces:
- gravitational force on the ball and girl
- normal force from the ground on girl



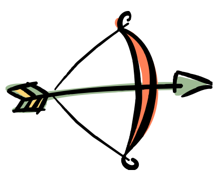
External forces applied to a system lead to a transfer of energy to or from the system. Internal forces within a system may lead to transformation of energy from one form to another. For example, in System A:

- Energy transfers: energy transferred due to gravitational force between girl and earth (external)
- Energy transformations: energy change due to the force of friction between slide and girl (internal).

Types of Energy/Energy Storage:

A. ***Potential Energy*** is energy stored in the arrangement of a physical system. Types of potential energy:

- *Gravitational Potential Energy* (E_g). For example, a bowling ball held at a vertical position above the ground before letting go of it (where the ground represents the zero height position), has gravitational potential energy – the ball has the potential to change its position with respect to the ground. When the ball is on the ground, it has no gravitational potential energy.



- *Elastic potential energy* (E_{el}). A drawn bow possesses elastic potential energy due to its stretched position with respect to its non-stretch position – the bow has the potential to change its form with respect to its initial form.
- *Chemical potential energy* (E_c). A chemical substance that undergoes a change in its molecular structure (the arrangement, or positioning, of atoms in the molecules that make up the substance is changing).

B. ***Kinetic Energy*** (E_k) is energy stored in the motion of a physical system. A moving car or a bouncing ball posses kinetic energy due to their motion. Kinetic energy of an object (physical system) can easily be transformed to other methods of energy storage/types of energy.

C. ***Thermal energy*** (E_{th}) is energy stored in the motion of the molecules that make up an object and it is connected to the microscopic structure of the object.

Mechanical energy is the energy which is possessed by an object due to its motion and its stored energy of position. Mechanical energy can be either energy of motion (kinetic energy) or stored energy of position (gravitational or elastic potential energy). Objects have mechanical energy if they are in motion and/or if they are at some position relative to a zero potential energy position. For example, a moving baseball possesses mechanical energy due to both its high speed (kinetic energy) and its vertical position above the ground (gravitational potential energy).

Energy Transformations:

Energy can also be transformed from one form of storage to another within the system. For example, a ball thrown vertically upward with an initial velocity initially has kinetic energy due to its motion (velocity). As the ball rises the velocity of the ball decreases and eventually becomes zero at the highest point. At that highest point, when the ball stops and is ready to reverse its motion, the ball has no kinetic energy (the velocity is zero). What happened to all that kinetic energy? Well, the ball changed its position with respect to the ground: it is now at a higher position and therefore it has gravitational potential energy. The kinetic energy of the ball did not disappear; it was actually transformed into gravitational potential energy.

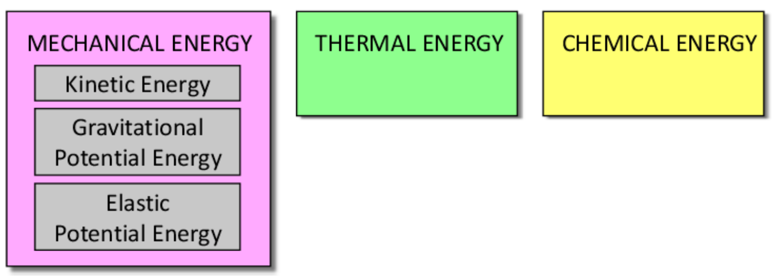
Energy Transfers:

Energy can be transferred in or out of a physical system through different transfer mechanisms:

1. ***working***: energy is transferred in or out of the system through forces that cause displacements (move something).

2. ***heating***: a temperature difference between two objects/physical systems in contact with each other always causes energy to be transferred from the warmer object to the colder object.

3. ***radiating***: objects/physical systems gain energy when radiation is absorbed and lose energy when radiation is emitted.



Reading Page: The Law of Conservation of Energy

One of the most fundamental laws of nature is the Conservation of Energy Law:

Regardless of the storage mechanism or the transfer mechanism, the total energy of a physical system is conserved.

or stated in a different way

Energy cannot be created or destroyed; it can only be transformed from one form of storage to another or transferred from one system to another.

This Law of Conservation of Energy can be applied to all physical systems. The only systems we will consider are mechanical systems (no chemical transformations). Mechanical systems can be isolated systems or non-isolated systems:

Isolated system: No energy is transferred into or out of the system. Each form of energy within the system can change (be transformed), but the total change in energy is zero. Energy of the system is conserved.

Non-isolated system: Energy can be exchanged with the environment through working, heating or radiating. A system is non-isolated if external forces act on it, if it is in contact with another system/environment at a different temperature or if radiation is absorbed or emitted. The energy of the system changes through one of the energy transfer methods. Working can transfer energy into or out of a system through an external force. Heating can transfer energy through contact between systems at different temperatures. Radiation can transfer energy through absorption or emission. The total energy (the energy of the system plus the energy of the environment) is conserved (stays the same).

Systems with mechanical and thermal energy only: The initial mechanical energy, plus the work done, equals the final mechanical energy plus additional thermal energy.

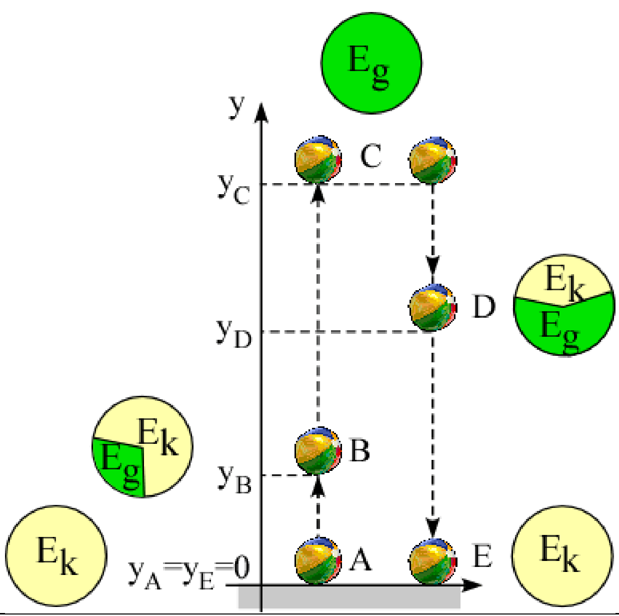
Reading Page: Using Pie Charts to Represent Energy

Pie charts are a very useful tool for representing energy of a system, energy transfers and energy transformations within the system. The total energy of the system is represented by a circle (pie) and the different storage mechanisms are represented by slices of the circle (slices of the pie). The examples below show you how to build such pie charts.

Example 1:
A ball is thrown up from ground level. Analyze the mechanical energy of the system at positions A, B, C, D, and E. What energy transformations take place? Is there any energy transfer to or from the environment (outside the system)? Ignore air resistance.

Solution:

System: The ball + Earth.
E_g = 0 on the ground



This system is an isolated system (no external forces acting on it), therefore no energy is transferred into or out of the system.

Position A:
$$E_{totalA} = E_k$$

Position B:
$$E_{totalB} = E_k + E_g$$

Position C:
$$E_{totalC} = E_g$$

Position D:
$$E_{totalD} = E_k + E_g$$

Position E:
$$E_{totalE} = E_k$$

At position **A** the only type of energy stored in the system is kinetic energy. Because the ball is on the ground, its gravitational potential energy is zero and there is no deformation in the ball therefore the elastic potential energy is also zero. Therefore, the total mechanical energy of motion of the ball is: $E_{totalA} = E_k$. The pie that represents total mechanical energy at position A is all made up of $\boldsymbol{E_k}$.

At position **B**, the relative arrangement of the ball and Earth has changed. The ball is higher above the ground and therefore, the gravitational potential energy of the system changed. Now the system has both kinetic (ball still moving) and gravitational potential energy. But there was no energy transfer from the outside of the system, therefore some of the kinetic energy of the ball must have transformed into gravitational potential energy (the ball is moving slower at this point, therefore it has less kinetic energy): there was an energy transformation within the system. The total mechanical energy of the system is made up of both kinetic and gravitational potential energy: $E_{totalB} = E_k + E_g$. The pie that represents the total mechanical energy at position B is part $\boldsymbol{E_k}$, part $\boldsymbol{E_g}$.

At position **C**, when the ball reaches the maximum height, it stops momentarily, and therefore the system has no more kinetic energy of motion: the total energy of the system is made up only of gravitational potential energy: $E_{totalC} = E_g$. The pie that represents the total mechanical energy at position C is all $\boldsymbol{E_g}$.

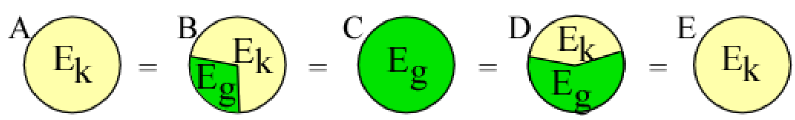
On its way down, at position **D**, the ball is moving and the relative arrangement of the ball and Earth has changed; some of the gravitational potential energy of the system is transformed back into kinetic energy. The total mechanical energy of the system is made up of both kinetic and gravitational potential energy: $E_{totalD} = E_k + E_g$. The pie that represents the total mechanical energy at position D is part $\boldsymbol{E_k}$, part $\boldsymbol{E_g}$.

How does one know that in this case gravitational potential energy decreased? Simple: no energy is transferred to or from the system, therefore some of the gravitational potential energy must have transformed into kinetic energy, and thus there is less gravitational potential energy stored in the system at position D than at position C.

At position **E**, at the instant before the ball hits the ground, the arrangement of the objects within the system is the same as for position A: the system has no more gravitational potential energy. It only has kinetic energy due to the motion of the ball before it hits the ground. The total mechanical energy of the system is: $E_{totalE} = E_k$. The pie that represents total mechanical energy is all $\boldsymbol{E_k}$ again.

Note: be careful, the kinetic energy of the ball at point E is not zero! The ball does not reach the ground with zero velocity. Have you ever seen a ball falling towards the ground and slowing down until it stops just before it hits the floor?

Notice how the pies always had the same size, only the "portion" (slice) of $\boldsymbol{E_k}$ and $\boldsymbol{E_g}$ changed. This shows that the total mechanical energy of the system is conserved. We can write the conservation of energy for this system at every point in different ways.

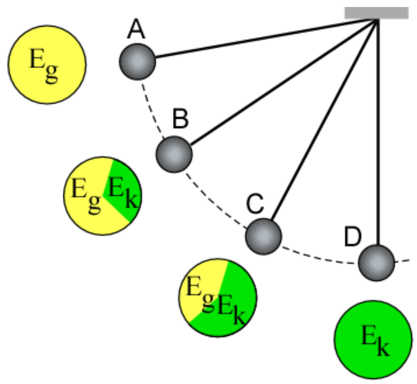


$$E_{kA} + \cancelto{0}{E_{gA}} = E_{kB} + E_{gB} = \cancelto{0}{E_{kC}} + E_{gC} = E_{kD} + E_{gD} = E_{kE} + \cancelto{0}{E_{gE}}$$

$$E_{kA} = E_{kB} + E_{gB} = E_{gC} = E_{kD} + E_{gD} = E_{kE}$$

<u>Example 2</u>:
A pendulum bob is pulled upward and when released it swings from its point of release to its lowest point. Analyze the total mechanical energy of the system at points A, B, C and D. Ignore air resistance.

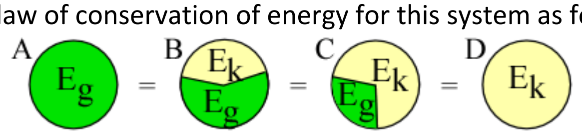
Solution:

System: pendulum bob + Earth $E_g = 0$ at position D  	This system is an isolated system (no external forces acting on it), therefore no energy is transferred into or out of the system and thus energy is conserved. Position A: $E_{totalA} = E_{gA} + \underbrace{E_{kA}}_{=0} = E_{gA}$ Position B: $E_{totalB} = E_{gB} + E_{kB}$ Position C: $E_{totalC} = E_{gC} + E_{kC}$ Position D: $E_{totalD} = \underbrace{E_{gD}}_{=0} + E_{kD} = E_{kD}$

As the pendulum is released (initial velocity is zero) and swings from its highest position A, to its lowest position with respect to the ground, position D, the arrangement of the objects within the system changes, thus the gravitational potential energy of the system changes from the highest value at A to zero at D.

After the pendulum is released at position A, its velocity increases and thus its kinetic energy increases. There are no external forces acting on your system, therefore the total mechanical energy of the system is conserved at every step: the sum of the kinetic and potential energy for the system is constant. The only process that takes place in the system is energy transformation, from gravitational potential energy to kinetic energy. When the pendulum reaches position D, all its gravitational potential energy is now transformed into kinetic energy.

We can represent/write the law of conservation of energy for this system as follows:



$$\cancelto{0}{E_{kA}} + E_{gA} = E_{kB} + E_{gB} = E_{kC} + E_{gC} = E_{kD} + \cancelto{0}{E_{gD}}$$

$$E_{gA} = E_{kB} + E_{gB} = E_{kC} + E_{gC} = E_{kD}$$

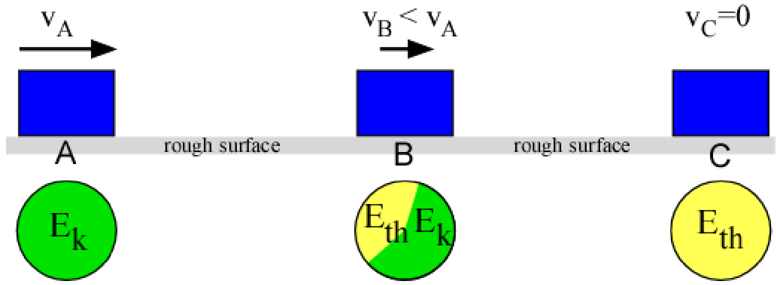
Example 3:

A heavy box is pushed such that it moves with a velocity v_A across a very rough floor (friction cannot be ignored). The force is removed. The box slides across the floor until it comes to a stop. Analyze the total mechanical energy of the box from the time when the force is removed to the time it stops.

Solution:

System: the box + Earth + ground.

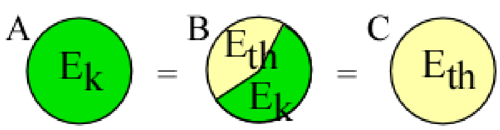
$E_g = 0$ on the ground



As the box slides across the floor, its vertical position with respect to the ground does not change, therefore the gravitational potential energy of the system does not change either, so it stays zero throughout. At position A, the entire energy of the system is kinetic energy. As the box slides along the floor, its speed is decreasing due to the friction between the box and floor. When it reaches position B, the box has a smaller speed than at A, therefore a smaller kinetic energy. But the total energy of the box + ground system is constant, which means that some of the initial kinetic energy must have transformed into a different type of energy. In this case, the bottom of the box and the ground surface both get warmer due to the friction between them so part of the kinetic energy is transformed into thermal energy. When the box reaches position C, it stops and all the kinetic energy has now been transformed into thermal energy.

Because the ground (earth) is part of our system, there is no energy transfer through heating with the environment, there are only energy transformations. If the system of study would have been the box only, then energy would have been transferred from the system to the environment through heating.

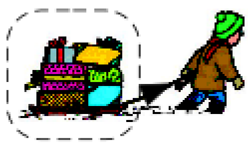
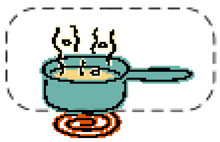
For the box + ground (earth) system we can write the conservation of energy as follows:

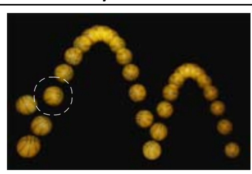
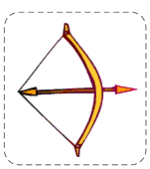
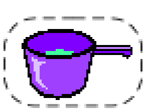
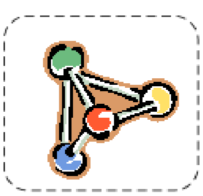
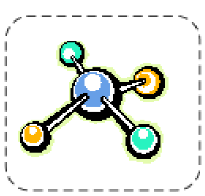


$$E_{kA} = E_{kB} + E_{thB} = E_{thC}$$

Analyzing work-heat-energy processes:

The table below allows you to quickly identify the type of mechanism for energy transfer and the type of energy in your system:

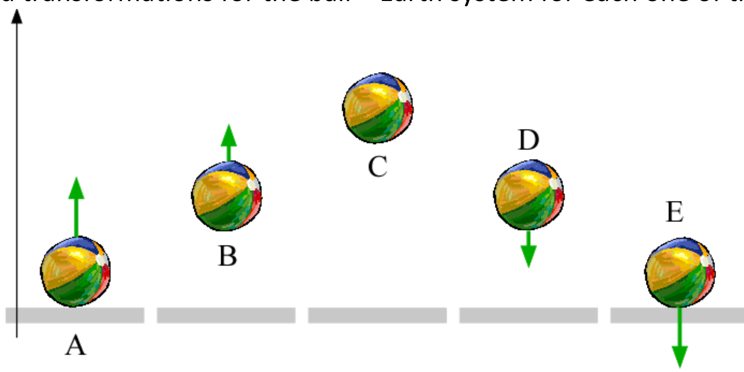
System	ENERGY TRANSFER MECHANISM
	Working Objects outside the system (the child) exert forces on objects inside the system (sled and boxes) as the system undergoes a displacement (moves from one point to another).
	Heating Object inside system (pot) touches another object outside the system (stove) that is at a different temperature.

System	ENERGY STORAGE MECHANISM
	Kinetic Energy, $\mathbf{E_k}$ Look for objects that are moving (have velocity).
	Gravitational Potential Energy, $\mathbf{E_g}$ Look for objects (the bowling ball) that change their position with respect to the ground.
	Elastic Potential Energy, $\mathbf{E_e}$ Look for an elastic object that is stretched or compressed.
 	Thermal Energy, $\mathbf{E_{th}}$ Look for a change in the temperature of an object or for a friction force that causes a thermal energy increase.
 	Chemical Energy, $\mathbf{E_c}$ Look for a change in the atomic, nuclear or molecular structure of an object.

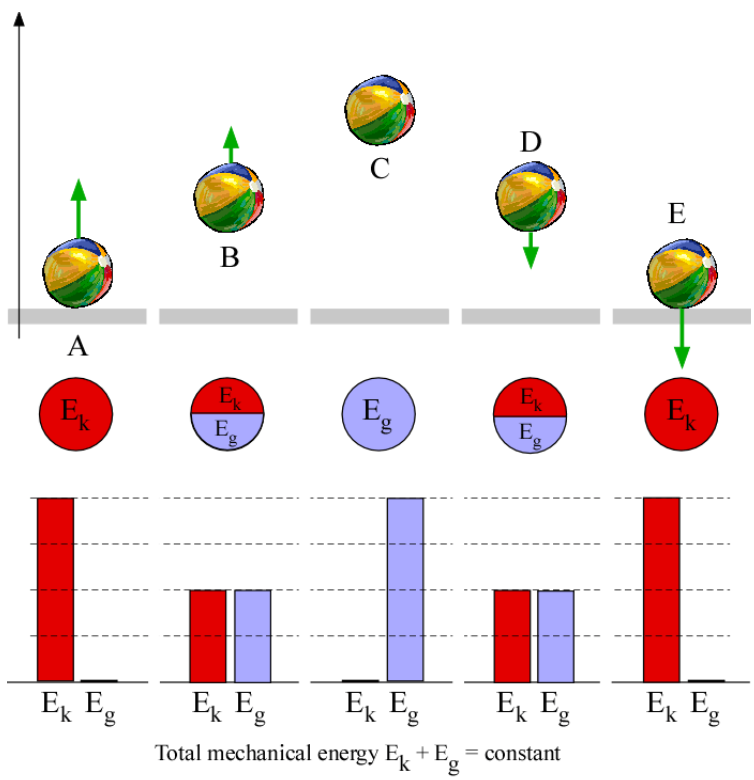
Reading Page: Using Energy Bar Graphs

Another way of representing energy transformations or energy transfers is through bar graphs instead of pie charts. Let us consider the examples below:

Example 1:
A ball is thrown upward from the ground level with an initial velocity (state A in the figure below). At point C it reaches its maximum height (its velocity is zero at this point) and it returns to the ground (point E). Analyze the energy transfers and transformations for the ball + Earth system for each one of the states (A through E).



Solution:
System: ball + earth.
This is a closed system (no external forces acting on it). Consider the ground as the zero of gravitational potential energy.



Position **A**: system has only kinetic energy (ball is moving).
Position **B**: system has both kinetic (ball moving) and gravitational potential energy (ball is above the ground).
Position **C**: system has only gravitational potential energy, the kinetic energy is zero (the ball stops at the highest point – has zero velocity at the highest point – while it is reversing its direction of motion).
Position **D**: system has both kinetic (ball moving) and gravitational potential energy (ball is above the ground).
Position **E**: system has only kinetic energy (ball is moving with its fastest speed when it hits the ground).

The system is closed (no external forces act on it) and there is no energy transfer between the system and the environment. During the entire process, the total mechanical energy of the system is conserved (remains constant). This is represented using the pie charts – size of the pie chart stays the same.

The energy bars (shown below each position of the ball) are a different representation of what type of energy the system has, how the energy is distributed in the system, how energy transforms from one form to another, and the fact that the sum of all those energies stays constant for an isolated system.

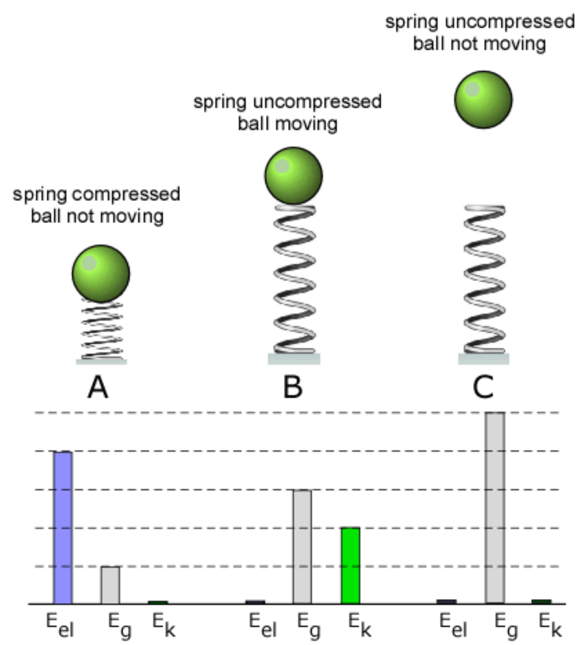
Mathematically, the conservation of energy that the bar graphs represent can be written as:

$$E_{totalA} = E_{totalB} = E_{totalC} = E_{totalD} = E_{totalE}$$
$$E_{kA} + \underbrace{E_{gA}}_{=0} = E_{kB} + E_{gB} = \underbrace{E_{kC}}_{=0} + E_{gC} = E_{kD} + E_{gD} = E_{kE} + \underbrace{E_{gE}}_{=0}$$
$$E_{kA} = E_{kB} + E_{gB} = E_{gC} = E_{kD} + E_{gD} = E_{kE}$$

There are no numerical scales on a bar chart but the bar heights should be drawn proportional to the amount of each type of energy and such that the sum of those energies stays constant. This example could be represented equally well through pie charts.

Example 2:
A ball is placed on a spring and pushed down such that the spring is compressed (position A). The ball is released. Analyze the energy transfers and transformations for the ball + spring + Earth system from the moment the ball is let go until it reaches the highest point and momentarily stops.

Solution:



System: ball+ spring + Earth.
This is a closed system (no external forces acting on it). The ground represents the zero of the gravitational potential energy.

Position **A**: system has no kinetic energy (ball is not moving). System has elastic potential energy (stored in the compressed spring) and gravitational potential energy (ball is above the ground).
Position **B**: system has both kinetic (ball is moving) and gravitational potential energy (ball is higher above the ground) but no more elastic potential energy (the spring is not compressed or stretched).
Position **C**: system has only gravitational potential energy, the kinetic energy is zero (the ball stops at the highest point before reversing its direction of motion) and the elastic potential energy is also zero (spring is not stretched or compressed).

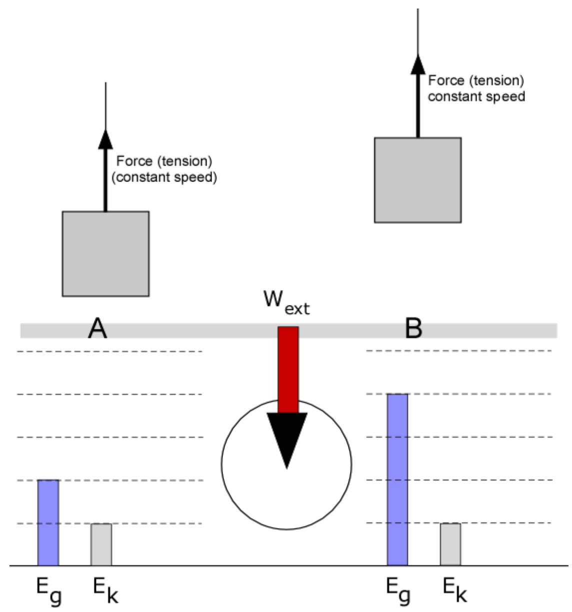
The energy bars below the three different positions show that energy is transformed from one form to another (you have different bars for different type of energies at different positions) but that the sum of those energies always stays the same (the sum of the height of those bars always stays constant). Mathematically, the conservation of energy for the three different positions is written as:

$$E_{totalA} = E_{totalB} = E_{totalC}$$
$$E_{eA} + E_{gA} + \underbrace{E_{kA}}_{=0} = \underbrace{E_{eB}}_{=0} + E_{gB} + E_{kB} = \underbrace{E_{eC}}_{=0} + E_{gC} + \underbrace{E_{kC}}_{=0}$$
$$E_{eA} + E_{gA} = E_{gB} + E_{kB} = E_{gC}$$

So far we have examined only isolated systems. Let's look at some examples where energy is transferred into or out of the system through working or heating.

<u>Example 3</u>:
A rope lifts a box off the ground at a constant speed. Analyze the energy transfers and transformations for the box + Earth system during the time the box is lifted at a certain height.

Solution:



System: box + Earth.
This is not a closed system because the rope, that exerts a force (tension force) on the box, is not part of the system. The ground represents the zero of the gravitational potential energy.

Position **A**: system has both kinetic energy (box is moving being lifted by the rope) and gravitational potential energy (box is above the ground).
Position **B**: system has both kinetic (box is moving) and gravitational potential energy (box is higher above the ground) but the kinetic energy at position B is the same as at position A because the velocity of the box did not change. The gravitational potential energy of the box increased because it is now higher above the ground.

If energy of the system is conserved, how is it possible to have more total energy than initially? The total mechanical energy of the system is still conserved but in this case one must also take into consideration the mechanism by which energy is transferred into the system. The tension in the rope does work on the box, thus increasing its gravitational potential energy. There is no energy transfer through heating for the system. The energy bars we have used above must be extended to include the energy transfer mechanism (working or heating).

Let's consider that at position A the system has 2 blocks of gravitational potential energy and one block of kinetic energy. At position B, the kinetic energy of the block is the same, one block, because its speed did not change but its potential energy is now larger. Let's suppose that the box has now 4 blocks of gravitational potential energy. The two additional blocks of energy were transferred into the system through working: the tension in the rope (an external force applied to the system) does work when lifting the box. This work is represented with an arrow of length (without the tip) equal to two blocks of energy and shown going into the system (circle). Thus it shows that this work is positive, and it transfers energy into the system.

The conservation of energy law written under the energy bar graphs accounts for both energy transfers to and from the system and energy transformations within this system. In this case, the conservation of energy law will be written as:

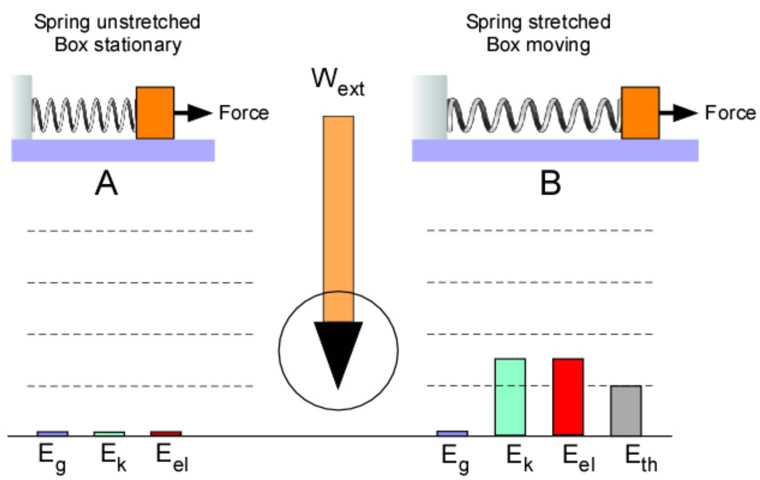
$$E_{totalA} + W_{ext} = E_{totalB}$$
$$E_{gA} + \underbrace{E_{kA}}_{=E_{kB}} + W_{ext} = E_{gB} + \underbrace{E_{kB}}_{=E_{kA}}$$

The sum of the height of all bar graphs (columns) for each state (including the work done on the system) must stay constant.

<u>Example 4:</u>
A box is attached to one end of a spring. The other end of the spring is anchored to the wall. Initially the box is at rest at the spring's equilibrium position (spring not stretched or compressed). A rope pulls with a force on the box across a very rough floor. Analyze the energy transfers and transformations for the spring + box + earth system during the time the box is pulled a certain distance to the right.

Solution:

System: spring + box + ground + earth.
This is not a closed system because the rope, that exerts a force (tension force) on the box, is not part of the system. The ground represents the zero of the gravitational potential energy.



Position **A**: system has no kinetic energy (box is not moving), no gravitational potential energy (box is on the ground) and no elastic potential energy (spring is not stretched or compressed). Therefore, the total mechanical energy of the system in position A is zero.
Position **B**: system has both kinetic (box is moving) and elastic potential energy (the spring is stretched) but no gravitational potential energy (box is still on the ground).
Where does this energy come from? It must be transferred either through working or heating. The force applied to the box is an external force and it transfers energy into the system through working. Let's assume that 4 blocks of energy are transferred into the system. The transferred energy is converted into kinetic energy (1 and a half block), elastic potential energy (one and a half block) and thermal energy (one block). Do not forget that because the ground is rough, there will be friction between the ground and the box as the box is moving across the ground. As a result, both the box and the ground get warmer, thus part of the energy in the system is thermal energy and it must be taken into consideration.

The conservation of energy law for this system will be written as:

$$E_{\text{total initial}} + W_{ext} = E_{\text{total final}} + E_{th}$$
$$\underbrace{E_g}_{=0} + \underbrace{E_k}_{=0} + \underbrace{E_e}_{=0} + W_{ext} = \underbrace{E_g}_{=0} + E_k + E_e + E_{th}$$

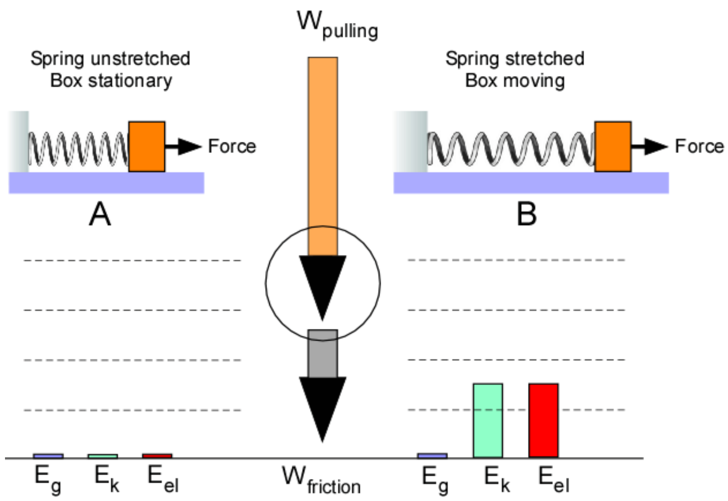
As always, the sum of the height of all bar graphs (columns) for each state (including the work done on the system) must stay constant.

Example 5:

Now let us consider the same system as in example 4 but we will not include the ground/floor in the system. We want to see how the energy bar graphs diagram changes.

Solution:

System: box + spring + earth.
The ground represents the zero of the gravitational potential energy.



Since the ground/floor is not part of our system it is important to remember that there is friction between the box and the floor and thus there is another external force, the friction force (beside the pulling force), acting on it. Due to the pulling force there will be a transfer of energy into the system through working. Due to the friction force there will be a transfer of energy out of the system also through working. These energy transfers can be added to the bar graphs as shown.

If the pulling force transfers four blocks of energy into the system, and one and a half is transformed into elastic potential energy and one and a half into kinetic energy, there is one block of energy left. This energy is transferred out of the system through working done by the force of friction. Note that the only difference between example D and example E is that in one case this block of energy is thermal energy and in the other case it is work done by the friction force. But the amount of thermal energy and work done by the friction force is the same.

In this case the conservation of energy law for this system will be written as:

$$E_{\text{total initial}} + W_{pulling} - W_{friction} = E_{\text{total final}} + E_{th}$$

$$\underbrace{E_g}_{=0} + \underbrace{E_k}_{=0} + \underbrace{E_{el}}_{=0} + W_{pulling} - W_{friction} = \underbrace{E_g}_{=0} + E_k + E_{el} + \underbrace{E_{th}}_{=0}$$

$$W_{pulling} - W_{friction} = E_k + E_{el}$$

Always try to make sure that you add or subtract work on one side and add the thermal energy on the other side.

Note: when drawing an energy bar graph, make sure you first draw all the energies in the initial state, and then draw all the energies in the final state. This will help you figure out if work is done to transfer energy into or out of the system.